Mr. G's little booklet on

Arithmetic and Geometric Series

Issue 5.0 6/16

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Mr. G's Little Booklets are

1. Symbols and Definitions
2. Circular Functions
3. Hyperbolic Functions
4. Complex Numbers
5. Calculus
6. Series
7. Venn Diagrams
8. Logic and Propositional Calculus
9. Vectors and Matrices
10. Probability
11. Laplace and Fourier Transforms
12. Miscellaneous Aspects of Mathematics
13. Statistical Tables
14. Trigonometric and Logarithmic Tables
15. Investigations - General
16. Investigations - Number

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Arithmetic Series

$u_1 + (u_1 + d) + (u_1 + 2d) + (u_1 + 3d) \ldots + u_n$

so we have four terms u_1, d, n and u_n. *Any three terms are sufficient to define the series*

so $u_n = u_1 + (n - 1)\, d$ *In IB formula book.*

and $u_1 = u_n - (n - 1)\, d$

and $d = (u_n - u_1) / (n - 1)$ *or in general* $(u_a - u_b) / (a - b)$

and $n = \{ (u_n - u_1) / d \} + 1$

Use the formulae above to find any term from the other three.

$S_n = \frac{1}{2} n\, (u_1 + u_n)$ *ie middle term x no. terms*

By substitution $S_n = \frac{1}{2} n \{ 2u_1 + (n - 1)\, d \}$ *In IB formula book.*

Geometric Series

$u_1 + u_1 r + u_1 r^2 + u_1 r^3 \ldots + u_n$

So again we have four terms u_1, r, n and u_n, *any three terms sufficient to define the series*

so $u_n = u_1 r^{(n - 1)}$

and $u_1 = u_n / r^{(n - 1)}$

and $r = \sqrt[(n-1)]{(u_n / u_1)}$ *the* $(n-1)^{th}$ *root of ratio* u_n / u_1

To find n by a formula is outside the IB Studies syllabus. Use trial and improvement

Students should create a simple series 1, 2, 4, 8 and identify u_1, u_n, *n and d*

Then use the formulae above to find any term from the other three. (except n)

$S_n = u_1(r^n - 1) / (r - 1)$ *when r > 1*

$S_n = u_1(1 - r^n) / (1 - r)$ *when r < 1*

If r < 1 it is possible to have an infinite series summing to a finite number.

$S_\infty = u_1 / (1 - r)$ *which is really neat*

Compound Interest

If a sum of money compounds by 3% per annum, this is a geometric series where

u_1 = *initial investment* r = 1.03 *in this case*

n = *number years of investment* u_n = *final value*

Binomial Series

$$(a+b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}b + {}^nC_1\, a^{n-2}b^2 + \ldots {}^nC_n\, b^n$$

where ${}^nC_r = {}^{n!}/_{(n-r)!\,r!}$ ${}^nC_r$ *also written* $\binom{n}{r}$

$$(1+x)^n = 1 + nx + {}^{n(n-1)x^2}/_{2!} + {}^{n(n-1)(n-2)x^3}/_{3!} + \ldots$$

$$a(1 + {}^b/_a x)^n = a\{1 + n({}^b/_a x) + {}^{n(n-1)(b/a\,x)^2}/_{2!} + {}^{n(n-1)(n-2)(b/a\,x)^3}/_{3!} + \ldots\}$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 \ldots$$

Power Series

$\sum r^0 = n$ *as* $r^0 = 1$

$\sum r^1 = {}^1/_2\, n\,(n+1)$

$\sum r^2 = {}^1/_6\, n\,(n+1)\,(2n+1)$ *see general term page 4*

$\sum r^3 = {}^1/_4\, n^2\,(n+1)^2 = [\sum r]^2$

$\sum r^4 = {}^1/_{30}\, n\,(n+1)(2n+1)(3n^2+3n-1)$

$\sum r^5 = {}^1/_{12}\, n^2\,(n+1)^2\,(2n^2+2n+1)$

$\sum r^6 = {}^1/_{42}\, n\,(n+1)(2n+1)\,(3n^4+6n^3-3n+1)$

$\sum r^7 = {}^1/_{24}\, n^2\,(n+1)^2\,(3n^4+6n^3-n^2-4n+2)$

Series Summary by Method of Differences

$u_r = f(r) - f(r-1)$ $\qquad f(r-1) - f(r)$

then $\sum u_r = f(n) - f(0)$ $\qquad f(0) - f(n)$

if $u_r = f(r) - f(r-k)$

then write out two series and all terms will still cancel except k leading and k trailing terms

General Solution for Summation of Algebraic/Power Series

The summation to n terms is given by

$$a_1(n)\,/\,1! + b_1(n)(n-1)\,/\,2! + c_1(n)(n-1)(n-2)\,/\,3!$$

where a_1 *is the* 1st *term,* b_1 *the* 1st *difference,* c_1 *the* 2nd *difference etc.*

Maclaurin's Series

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ldots$$

by successive differentiation and setting x = 0 each time we derive

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) \ldots$$

Taylor's Series

$$f(x + a) = f(a) + x f'(a) + \frac{x^2}{2!} f''(a) + \frac{x^3}{3!} f'''(a) \ldots$$

$$f(x) = f(a) + (x-a) f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) \ldots$$

Exponential Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \ldots \quad \dagger$$

hence $e = 1 + 1 + \frac{1}{2!} + \frac{1}{3!} \ldots$

$$a^x = e^{x \ln a} = 1 + x \ln a + \frac{(x \ln a)^2}{2!} + \frac{(x \ln a)^3}{3!} \ldots$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots \quad {}^{-}1 < x \le {}^{+}1$$

hence $\ln 2 = x - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \ldots$

Miscellaneous Series

$$2 = \frac{1}{1} + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \ldots$$

$$2 = 1 + \frac{1}{2\times1} + \frac{1}{3\times2} + \frac{1}{4\times3} + \frac{1}{5\times4} + \frac{1}{6\times5} + \ldots$$

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \ldots \quad \ddagger$$

$$\frac{\pi^2}{90} = 1 + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \ldots \quad \ddagger$$

$$-\ln(1-p) = p + p^2/2 + p^3/3 + p^4/4 \ldots$$ *logarithmic distr.*

$$5^2 = 3^2 + 4^2$$

$$6^3 = 3^3 + 4^3 + 5^3$$

$$5^4 = 2^4 + 2^4 + 3^4 + 4^4 + 4^4$$

Notes

† *of course this can be deduced directly from Maclaurin's Series.*

‡ *examples of Riemann's zeta functions (discovered by Euler)*

Trigonometric Series

$\sin x = x - {}^{x^3}/_{3!} + {}^{x^5}/_{5!} - {}^{x^7}/_{7!} \ldots \qquad x \in P$

$\cos x = 1 - {}^{x^2}/_{2!} + {}^{x^4}/_{4!} - {}^{x^6}/_{6!} \ldots \qquad x \in P$

$\tan x = \{{}^{2^2(2^2-1)}/_{2!}\}B_2x - \{{}^{2^4(2^4-1)}/_{4!}\}B_4x^3 + \{{}^{2^6(2^6-1)}/_{6!}\}B_6x^5 - \ldots$

$= {}^{1x}/_{1!} + {}^{2x^3}/_{3!} + {}^{16x^5}/_{5!} + {}^{272x^7}/_{7!} + {}^{7936x^9}/_{9!} + {}^{353792x^{11}}/_{11!} \cdots$

$= x + {}^{x^3}/_3 + {}^{2x^5}/_{15} + {}^{17x^7}/_{315} + {}^{62x^9}/_{2835} \cdots \quad -\tfrac{1}{2}\pi \le x \le \tfrac{1}{2}\pi$

or as a continuous fraction $x/(1-(x^2/3-(x^2/5-(x^2/7-(x^2/9\ldots$

in practice taking 5 terms will give a reasonable approximation for $x \le 1$ *only*

$\sin^{-1} x = x + {}^{1x^3}/_{2.3} + {}^{1.3.x^5}/_{2.4.5} + {}^{1.3.5.x^7}/_{2.4.6.7} \cdots$

$\cos^{-1} x = \tfrac{1}{2}\pi - \sin^{-1} x$

$\tan^{-1} x = x - {}^{x^3}/_3 + {}^{x^5}/_5 - {}^{x^7}/_7 \ldots \qquad |x| \le 1$

$\sinh x = x + {}^{x^3}/_{3!} + {}^{x^5}/_{5!} \ldots \qquad x \in P$

$\cosh x = 1 + {}^{x^2}/_{2!} + {}^{x^4}/_{4!} \ldots \qquad x \in P$

$\tanh x = x - {}^{x^3}/_3 + {}^{2x^5}/_{15} - {}^{17x^7}/_{315} \cdots$

$\sinh^{-1} x = x - {}^{1x^3}/_{2.3} + {}^{1.3.x^5}/_{2.4.5} - {}^{1.3.5.x^7}/_{2.4.6.7} \cdots$

$\cosh^{-1} x = \ln 2x - \{{}^{1.x^{-2}}/_{2.2} + {}^{1.3.x^{-4}}/_{2.4.4} - {}^{1.3.5.x^{-6}}/_{2.4.6.6} \cdots\}$

$\tanh^{-1} x = x + {}^{x^3}/_3 + {}^{x^5}/_5 + {}^{x^7}/_7 \ldots$

Bernoulli Numbers

$B_0 = 1$, $B_1 = {}^{-1}/_2$, $B_2 = {}^1/_6$, $B_3 = 0$, $B_4 = {}^{-1}/_{30}$, $B_5 = 0$, $B_6 = {}^1/_{42}$, $B_7 = 0$, $B_8 = {}^{-1}/_{30}$,
$B_{10} = {}^5/_{66}$, $B_{12} = -{}^{691}/_{2730}$, $B_{14} = {}^7/_6$, $B_{16} = -{}^{3617}/_{510}$, $B_{18} = {}^{43867}/_{792}$

Harmonic Series

$H_n = {}_{r=1}\Sigma^n\, {}^1/_r = 1 + {}^1/_2 + {}^1/_3 + {}^1/_4 + \ldots {}^1/_n$

$\approx \ln n + \gamma + {}^1/_{2n}$

where γ *is the Euler-Mascheroni constant itself defined by* $\text{limit}_{n\to 0}\, H_n - \ln n$

Notes

small angle approximations where $\theta \approx 0$

$\sin\theta \approx \theta$

$\cos\theta \approx 1 - \tfrac{1}{2}\theta^2$

$\tan\theta \approx \theta$

Other Series

$$(1 + x)^{-1} \qquad 1 - x + x^2 - x^3 + x^4 \ldots$$

$$^1/_{1+x^2} = 1 - x^2 + x^4 - x^6 + x^8 \ldots$$ *A continuous function*

Although LHS function is continuous, RHS partial sums only converge for $|x| < 1$

$$^1/_{1-x^2} = 1 + x^2 + x^4 + x^6 + x^8 \ldots$$ *This has discontinuities*

Summation of some common series

$$\sum r = {}^1/_2\, n(n + 1)$$

$$\sum r(r + 1) = {}^1/_3\, n(n + 1)(n + 2) \qquad \dagger$$

$$\sum r(r + 1)(r + 2) = {}^1/_4\, n(n + 1)(n + 2)(n + 3)$$

$$\sum r(r+1)(r+2)\ldots(r+k) = {}^1/_{(k+2)}\, n(n + 1)(n + 2)\ldots(n + k + 1)$$

$$\sum r(r + 0) \equiv \sum r^2 = {}^1/_6\, n(n + 1)(2n + 1)$$

$$\sum r(r + 1) = {}^1/_6\, n(n + 1)(2n + 4)$$ *obviously same as* †

$$\sum r(r + 2) = {}^1/_6\, n(n + 1)(2n + 7)$$

$$\sum r(r + k) = {}^1/_6\, n(n + 1)(2n + 3k + 1)$$

$$\sum {}^1/_{r(r+1)} = {}^n/_{1(n+1)} = 1 - {}^1/_{(n+1)}$$

$$\sum {}^1/_{(r+1)(r+2)} = {}^n/_{2(n+2)}$$ *given by* $^{(n+1)}/_{1(n+2)} - {}^1/_{1.2}$

$$\sum {}^1/_{(r+2)(r+3)} = {}^n/_{3(n+3)}$$ *given by* $^{(n+1)}/_{2(n+3)} - {}^1/_{2.3}$

$$\sum {}^1/_{(r+k)(r+k+1)} = {}^n/_{(k+1)(n+k+1)}$$ *cf* $^{(n+1)}/_{k(n+k+1)} - {}^1/_{k.(k+1)}$

$$\sum {}^1/_{r(r+1)(r+2)} = {}^{n(n+3)}/_{4(n+1)(n+2)} = {}^1/_4 - {}^1/_{2(n+1)(n+2)}$$

$$\sum {}^1/_{(r+1)(r+2)(r+3)} = {}^{n(n+5)}/_{12(n+2)(n+3)}$$

$$\sum {}^1/_{(r+2)(r+3)(r+4)} = {}^{n(n+7)}/_{24(n+3)(n+4)}$$

$$\sum {}^1/_{(r+k-1)(r+k)(r+k+1)} = {}^{n(n+2k+1)}/_{2n(n+1)(n+k)(n+k+1)}$$

$$\sum {}^1/_{r(r+1)(r+2)\ldots(r+k)} = {}^1/_{k \times k!} - {}^1/_{k(n+1)(n+2)(n+3)\ldots(n+k)}$$

This expression derived by the author. See separate investigation.

Arithmetic-Geometric Series

$$a + (a + d)r + (a + 2d)r^2 + \ldots = {}^a/_{(1-r)} + {}^{rd}/_{(1-r)^2} \qquad {}^-1 < r < {}^+1$$

$$S_n = {}^{a(1-r^\wedge n)}/_{1-r} + {}^{rd(1-r^\wedge(n-1))}/_{(1-r^2)} - {}^{(n-1)dr^\wedge n}/_{(1-r)}$$

Series for e

Define e as the limit of $(1 + {}^{1}/_{n})^{n}$. *First expand the expression by binomial.*

$$(a + b)^n \;=\; a^n + n\,a^{n-1}b/1! + n(n-1)a^{n-2}b^2/2! + n(n-1)n-2)a^{n-3}b^3/3! + \ldots$$

Put $a = 1$ *and* $b = {}^{1}/_{n}$

$$(1 + {}^{1}/_{n})^n \;=\; 1 + 1 + {}^{1}/_{2!}({}^{n-1}/_{n}) + {}^{1}/_{3!}\{{}^{(n-1)(n-2)}/_{n^2}\} + {}^{1}/_{4!}\{{}^{(n-1)(n-2)(n-3)}/_{n^3}\} \ldots$$

As $n \rightarrow \infty$ *then* ${}^{n-1}/_{n}$, ${}^{(n-1)(n-2)}/_{n^2}$ *etc.* $\rightarrow 1$

$$e \;=\; {}^{1^0}/_{0!} + {}^{1^1}/_{1!} + {}^{1^2}/_{2!} + {}^{1^3}/_{3!} \ldots$$

By the same method we can also deduce

$$e^x \;=\; 1 + {}^{x^1}/_{1!} + {}^{x^2}/_{2!} + {}^{x^3}/_{3!} \ldots$$

$$e^{-1} \;=\; {}^{1}/_{0!} - {}^{1}/_{1!} + {}^{1}/_{2!} - {}^{1}/_{3!} \ldots$$ *immediately follows*

Alternative Representation of Series

n^{th} term	$u_1\ u_2\ u_3$	u_{n+1}
$n + 2$	3, 4, 5,	$u_n + 1$
$2n - 1$	1, 3, 5,	$u_n + 2$
$3n + 2$	5, 8, 11,	$u_n + 3$
$\frac{1}{2}n + \frac{1}{2}$	1, 1½, 2,	$u_n + \frac{1}{2}$
$an + b$	a+b, 2a+b, 3a+b	$u_n + a$
n^2	1, 4, 9,	$u_n + 2n + 1$

Notes

Here is Goodhand's method for a faster approximation to e

$$e \qquad \lim_{n\rightarrow\infty} \tfrac{1}{2}\{(1 + {}^{1}/_{n})^n + (1 - {}^{1}/_{n})^{-n}\}$$ *correct to 4dp for* $n=136$

In subsequent series differentiating LHS and RHS will give you further series. Remember

limit $x\rightarrow 0\ {}^{\sin x}/_{x} = 1$ *limit* $x\rightarrow 0\ {}^{\cos x - 1}/_{x} = 0$ *limit* $x\rightarrow 0\ {}^{\tan x}/_{x} = 1$

L'Hospital's rule $\lim_{x\rightarrow a} {}^{f(x)}/_{g(x)} = \lim_{x\rightarrow a} {}^{f'(x)}/_{g'(x)}$

Series of e and π

$$e^{-1} = {}^{1}/_{0!} - {}^{1}/_{1!} + {}^{1}/_{2!} - {}^{1}/_{3!} \ldots$$

$$e = {}^{1}/_{0!} + {}^{1}/_{1!} + {}^{1}/_{2!} + {}^{1^3}/_{3!} \ldots$$

$$e^{r} = {}^{r^0}/_{0!} + {}^{r^1}/_{1!} + {}^{r^2}/_{2!} + {}^{r^3}/_{3!} \ldots$$

because $e^{i\pi} = {}^{-}1$

$$1 = {}^{\pi^0}/_{0!} + {}^{\pi^1}/_{1!} + {}^{\pi^2}/_{2!} + {}^{\pi^3}/_{3!} \ldots$$

$${}^{-}1 = {}^{\pi^0}/_{0!} - {}^{\pi^2}/_{1!} + {}^{\pi\wedge 4}/_{2!} - {}^{\pi\wedge 6}/_{3!} \ldots$$

note also this tells us that π *is the imaginary part of the natural logarithm of (*${}^{-}1$*)*

$${}^{\pi}/_{4} = {}^{1}/_{1} - {}^{1}/_{3} + {}^{1}/_{5} - {}^{1}/_{7} + {}^{1}/_{9} - {}^{1}/_{11} \ldots$$

$${}^{\pi\wedge 3}/_{16\times 2!} = {}^{1}/_{1\wedge 3} - {}^{1}/_{3\wedge 3} + {}^{1}/_{5\wedge 3} - {}^{1}/_{7\wedge 3} + {}^{1}/_{9\wedge 3} - {}^{1}/_{11\wedge 3} \ldots$$

$${}^{5\pi\wedge 5}/_{64\times 4!} = {}^{1}/_{1\wedge 5} - {}^{1}/_{3\wedge 5} + {}^{1}/_{5\wedge 5} - {}^{1}/_{7\wedge 5} + {}^{1}/_{9\wedge 5} - {}^{1}/_{11\wedge 5} \ldots$$

$${}^{61\pi\wedge 7}/_{256\times 6!} = {}^{1}/_{1\wedge 7} - {}^{1}/_{3\wedge 7} + {}^{1}/_{5\wedge 7} - {}^{1}/_{7\wedge 7} + {}^{1}/_{9\wedge 7} - {}^{1}/_{11\wedge 7} \ldots$$

$${}^{1385\pi\wedge 9}/_{1024\times 8!} = {}^{1}/_{1\wedge 9} - {}^{1}/_{3\wedge 9} + {}^{1}/_{5\wedge 9} - {}^{1}/_{7\wedge 9} + {}^{1}/_{9\wedge 9} - {}^{1}/_{11\wedge 9} \ldots$$

$${}^{50521\pi\wedge 11}/_{4096\times 10!} = {}^{1}/_{1\wedge 11} - {}^{1}/_{3\wedge 11} + {}^{1}/_{5\wedge 11} - {}^{1}/_{7\wedge 11} + {}^{1}/_{9\wedge 11} - {}^{1}/_{11\wedge 11} \ldots$$

the Euler numbers are

1, 1,5, 61, 1385, 50521, 2702765, 199360981, 19391512145, 2404879675441…

and (possibly?) are defined by

$${}^{1}/_{\cos x} = {}^{1}/_{0!} + {}^{x^2}/_{2!} + {}^{5x\wedge 4}/_{4!} + {}^{61x\wedge 6}/_{6!} + {}^{1385x\wedge 8}/_{8!} \ldots$$

Probabilities and Primes (discovered by Euler)

$${}^{\pi^2}/_{6} = {}^{1}/_{1^2} + {}^{1}/_{2^2} + {}^{1}/_{3^2} + {}^{1}/_{4^2} \ldots =$$

$$(1+{}^{1}/_{2^2}+{}^{1}/_{4^2}+{}^{1}/_{8^2} \ldots)+(1+{}^{1}/_{3^2}+{}^{1}/_{9^2}+{}^{1}/_{27^2} \ldots)+(1+{}^{1}/_{5^2}+{}^{1}/_{25^2}+{}^{1}/_{125^2} \ldots)+$$

and so on through the primes. Then invert and use formula for geometric infinite series

$${}^{6}/_{\pi^2} = (1-{}^{1}/_{2^2})\times(1-{}^{1}/_{3^2})\times(1-{}^{1}/_{5^2})\times(1-{}^{1}/_{7^2})\times(1-{}^{1}/_{11^2}) \ldots$$

$${}^{6}/_{\pi^2} = {}^{3}/_{4} \times {}^{8}/_{9} \times {}^{24}/_{25} \times {}^{48}/_{49} \times {}^{120}/_{121} \times \ldots$$

Imagine picking two numbers at random.

The 1 st factor is ***P(not both divisible by 2)*** *2 nd factor* ***P(not both divisible by 3)*** *etc.*

Whole expression is probability two randomly chosen integers have no prime in common

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Greek Alphabet				Principle/Simplest Use	English	Type
alpha	Α	*not used*	α	*first root of quadratic*	a	**a**
beta	Β	*Beta function*	β	*second root of quadratic*	b	**b**
gamma	Γ	*Gamma function*	γ	*Euler's constant*	g	**g**
delta	Δ	*Difference operator*	δ	*small increment*	d	**d**
epsilon	Ε	*not used*	ε	*error*	short e	**e**
zeta	Ζ	*not used*	ζ	*Riemann zeta function*	z	**z**
eta	Η	*not used*	η	*efficiency*	long e	**h**
theta	Θ	*asymp. tight bound*	θ	*angle*	th	**q**
iota	Ι	*not used*	ι	*imaginary unit*	i	**i**
kappa	Κ	*not used*	κ	*curvature*	k	**k**
lambda	Λ	*diag. matrix eigen-values*	λ	*failure rate*	l	**l**
mu	Μ	*not used*	μ	*population mean*	m	**m**
nu	Ν	*not used*	ν	*poisson ratio*	n	**n**
xi	Ξ	*grand canonical ensemble*	ξ	*damping coefficient*	x	**x**
omicron	Ο	*limiting behaviour function*	ο	*generally not used*	short o	**o**
pi	Π	*Product operator*	π	*ratio* $^c/_d$ *circle*	p	**p**
rho	Ρ	*not used*	ρ	*correlation coefficient*	r	**r**
sigma	Σ	*summation*	σ	*standard deviation*	s	**s**
tau	Τ	*not used*	τ	*mean lifetime*	t	**t**
upsilon	Υ	*Bessel function*	υ	*generally not used*	u	**u**
phi	Φ	*cumulative function*	ϕ	*golden ratio*	ph	**f**
phi (alt.)	ϑ	*not used*	φ	**normal function** **scalar potential**	ph	**j**
chi	Χ	*probability function*	χ^2	*chi-squared prob.function*	ch	**c**
psi	Ψ	*not used*	ψ	*wave function*	ps	**y**
omega	Ω	*mathematical constant*	ω	*angular frequency*	long o	**w**
stigma	ς					**v**
pomega			ϖ	*angular velocity*		**v**

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Orders of Magnitude

septillionth	yocto-	y	10^{-24}	septillion	yotta-	Y	10^{24}
sextillionth	zepto-	z	10^{-21}	sextillion	zetta-	Z	10^{21}
quintillionth	atto-	a	10^{-18}	quintillion	exa-	E	10^{18}
quadrillionth	femto-	f	10^{-15}	quadrillion	peta-	P	10^{15}
trillionth	pico-	p	10^{-12}	trillion	tera-	T	10^{12}
billionth	nano-	n	10^{-9}	billion	giga-	G	10^{9}
millionth	micro-	µ	10^{-6}	million	mega-	M	10^{6}
thousandth	milli-	m	10^{-3}	thousand	kilo-	k	10^{3}
hundredth	centi-	c	10^{-2}	hundred	hecto-	h	10^{2}
tenth	deci-	d	10^{-1}	ten	deca-	da	10^{1}
one	-	-	10^{0}	one	-	-	10^{0}

Mathematical Constants - 30 decimals (last place not rounded)

pi	π	=	*3.14159 26535 89793 23846 26433 83279…*
exponential	e	=	*2.71828 18284 59045 23536 02874 71352…*
Pythagoras's	$\sqrt{2}$	=	*1.41421 35623 73095 04880 16887 24209…*
	$\sqrt{3}$	=	*1.73205 08075 68877 29352 74463 41505…*
	$\log 2$	=	*0.69314 71805 59945 30941 72321 21458…*
golden ratio	ϕ	=	*1.61803 39887 49894 84820 45868 34365…*
Euler-Mascheroni	γ	=	*0.57721 56649 01532 86060 65120 90082…*
Feigenbaum's	δ	=	*4.66920 16091 02990 67185 32038 20466…*
	$\xi(2)$	=	*1.64493 40668 48226 43647 24151 66646…*
Apery's	$\xi(3)$	=	*1.20205 69031 59594 28539 97381 61511…*
	$\xi(4)$	=	*1.08232 32337 11138 19151 60036 96541…*
Euler's	$\xi(5)$	=	*1.03692 77551 43369 92633 13654 86457…*
	$\xi(6)$	=	*1.01734 30619 84449 13971 45179 29790…*
	e^{π}	=	*23.14069 26327 79269 00572 90863 67948…*

Counting

No.	**Greek**	**Latin**
1	*mono*	*uni*
2	*duo*	*bi*
3	*tri*	*tri*
4	*tetra*	*quad*
5	*penta*	*quin*
6	*hexa*	*sex*
7	*hepta*	*sept*
8	*octo*	*oct*
9	*nona*	*non*
10	*deca*	*dec*

These booklets are written and produced by Robert Goodhand

Although the formulae and expressions given have been individually derived and checked errors do creep in. The booklets are also continuously updated.

If you would like the latest issue, just email me at robert.goodhand@gmail.com