

Mr. G's little booklet on

Venn Diagrams

Issue 4.2

7/16

 *rg*

Mr. G's Little Booklets are

- 1 Symbols and Definitions**
- 2 Circular Functions**
- 3 Hyperbolic Functions**
- 4 Complex Numbers**
- 5 Calculus**
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- 7 Venn Diagrams**
- 8 Logic and Propositional Calculus**
- 9 Vectors and Matrices**
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- 12 Miscellaneous Aspects of Mathematics**
- 13 Statistical Tables**
- 14 Trigonometric and Logarithmic Tables**
- 15 Investigations - General**
- 16 Investigations - Number**

Venn Diagrams I

$P \cap Q$ say P **intersection** Q

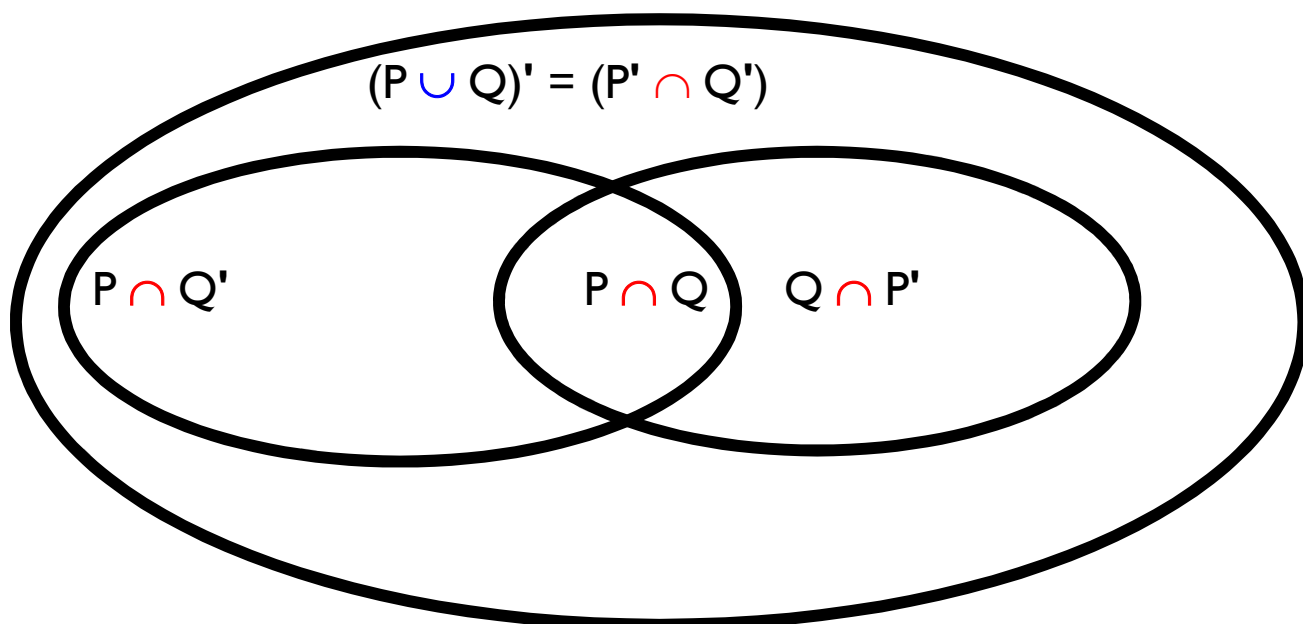
means P **AND** Q

$P \cup Q$ say P **union** Q

means P **OR** Q **or both**

$$n(P \cup Q)$$

$$= n(P) + n(Q) - n(P \cap Q)$$



where the left ellipse is (P) and the right ellipse is (Q)

Thus $n(P) = n(P \cap Q') + n(P \cap Q)$

and $n(Q) = n(Q \cap P') + n(Q \cap P)$

hence $n(P \cap Q) = n(P) - n(P \cap Q')$

$$n(P \cap Q) = n(Q) - n(Q \cap P')$$

From diagram

$$(P \cup Q)' = (P' \cap Q') \quad \dagger$$

$$(P \cap Q)' = (P' \cup Q') \quad \dagger$$

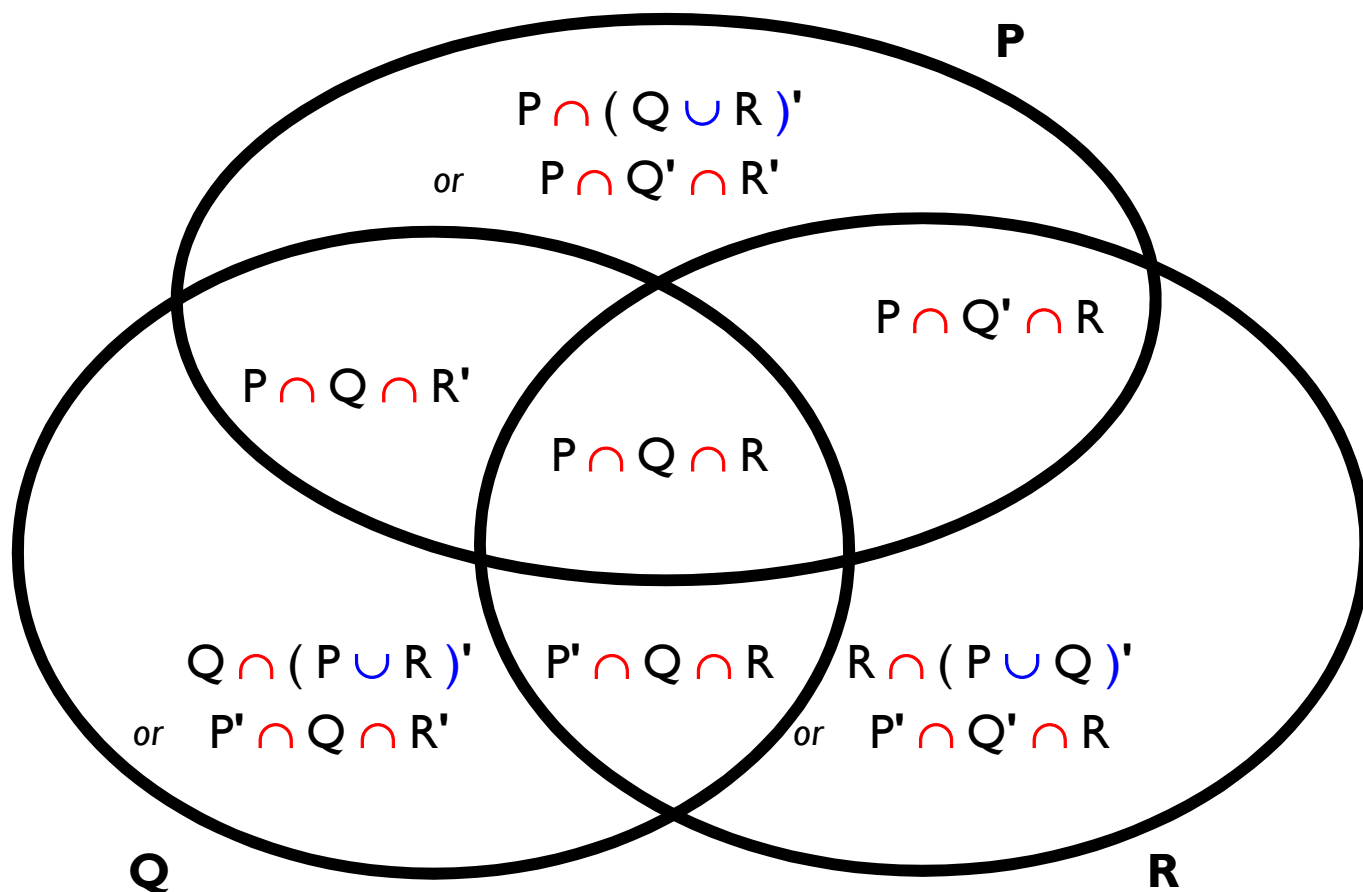
Notes

There is also the term $P \setminus Q$ (say "P diff Q") $\equiv P \cap Q'$

It immediately follows that $P \setminus Q \equiv Q' \setminus P'$ which is similar to the contrapositive rule.

† these are de Morgan's Rules

Venn Diagrams 2



$$n(P \cup Q \cup R) = n(P) + n(Q) + n(R) - n(P \cap Q) - n(P \cap R) - n(Q \cap R) + n(P \cap Q \cap R)$$

Other Set Relationships

if $(A \cap B) = B$ then $B \subseteq A$

or $(A \cup B) = A$ then $B \subseteq A$

if $(A \cap B) = A$ then $A \subseteq B$

or $(A \cup B) = B$ then $A \subseteq B$

Notes

Venn Diagrams

Truth Tables

Logic Gates

\cup union

\cap intersection *de Morgan*

T T F F
T F T F

p
q

\vee OR $\underline{\vee}$ XOR
 \wedge AND $\underline{\wedge}$ XAND

	U	ϵ
	$(P \cup Q)$	$(P' \cap Q)'$
	$(P \cup Q')$	$(P' \cap Q)'$
	P	
	$(P' \cup Q)$	$(P \cap Q)'$
	Q	
	$(P \cap Q) \cup (P' \cap Q')$	
	$(P' \cup Q)'$	$P \cap Q$
	$(P' \cup Q)$	$(P \cap Q)'$
	$(P \cap Q) \cup (P' \cap Q)$	
	Q'	
	$(P' \cup Q)'$	$P \cap Q'$
	P'	
	$(P \cup Q)'$	$P' \cap Q$
	$(P \cup Q)$	$P' \cap Q'$
	\emptyset	

T T T T
T T T F
T T F T
T T F F
T F T T
T F T F
T F F T
T F F F
F T T T
F T T F
F T F T
F T F F
F F T T
F F T F
F F F T
F F F F

truth
incl. disjunction †
implication
p
implication
q
equivalence/ IFF
conjunction
incompatible
excl. disjunction
negation
non implication
negation
non implication
joint denial
contradiction

$p \vee \neg p$ $\neg(p \wedge \neg p)$
 $p \vee q$ $\neg(\neg p \wedge \neg q)$
 $p \vee \neg q$ $\neg(\neg p \wedge q)$
 $p \vee p$ $\neg(\neg p \wedge \neg p)$
 $\neg p \vee q$ $\neg(p \wedge \neg q)$
 $q \vee q$ $\neg(\neg p \wedge \neg q)$
 $\neg(p \underline{\vee} q)$ $p \underline{\wedge} q$
 $\neg(\neg p \underline{\vee} \neg q)$ $p \wedge q$
 $\neg p \underline{\vee} \neg q$ $\neg(p \wedge q)$
 $p \underline{\vee} q$ $\neg(p \underline{\wedge} q)$
 $\neg(q \underline{\vee} q)$ $(\neg q \wedge \neg q)$
 $\neg(\neg p \underline{\vee} q)$ $p \wedge \neg q$
 $\neg(p \underline{\vee} p)$ $\neg p \wedge \neg p$
 $\neg(p \underline{\vee} \neg q)$ $\neg p \wedge q$
 $\neg(p \underline{\vee} q)$ $\neg p \wedge \neg q$
 $\neg(p \underline{\vee} \neg p)$ $(\neg p \wedge p)$

Notes

† also termed "exhaustiveness"

$\neg(p \vee q)$ is the same as $(\neg p \wedge \neg q)$ (everything gets multiplied by "¬")

If you look carefully, the colouring follows the Truth table exactly.

1st column colours the intersection

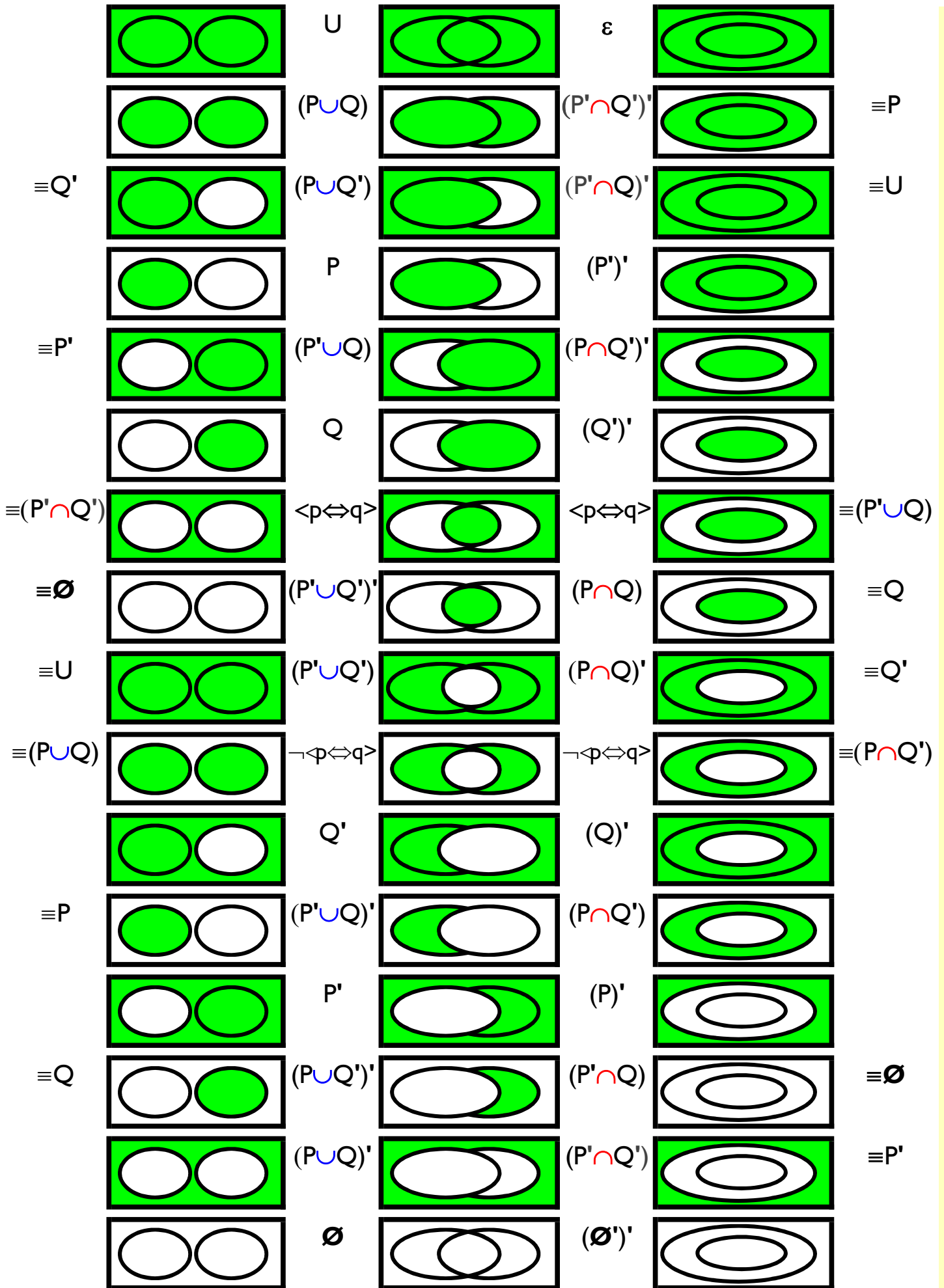
2nd column colours P

3rd column colours Q

4th column colours the outside

so $P \cap Q$, $P \cap Q'$, Q , $P' \cap Q$, $P' \cap Q'$ are coloured for T in each column in turn.

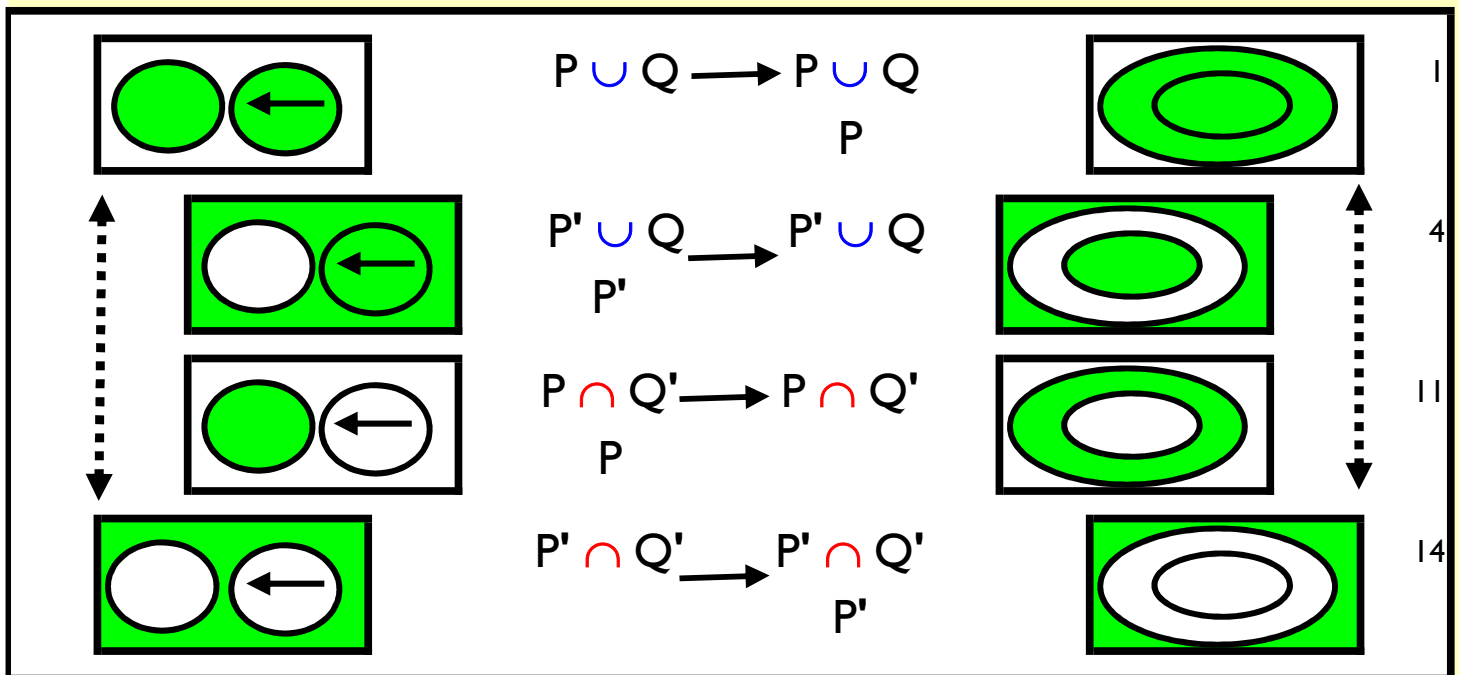
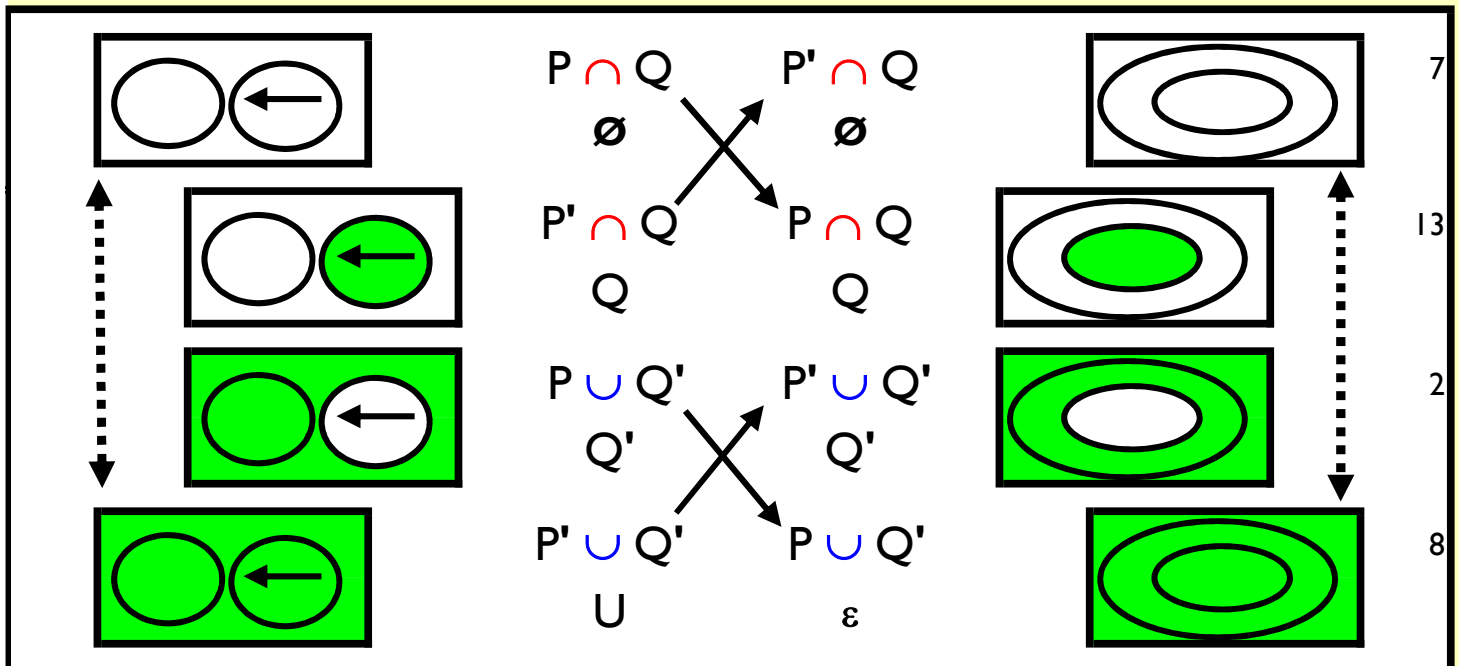
Venn Diagrams for disjunction, intersection and inclusion



Visual Representation of de Goodhand's Rule

To move between disjoint and inclusive sets, change P to P' (or P' to P)

for the set relationships that cannot be defined by P (or P') alone.



Notes

de Goodhand's rule holds for the top two pairs with their complements via de Morgan's rules

The bottom two pairs with complements are more self evident and pair without change.

Drawing Venn diagrams for disjoint and inclusion reduces the arrangements to 2^3 .

There are likewise 8 ways of writing an expression with three elements.

So there is a one-to-one pairing between the two and some pattern must then be apparent.

Counting

No.	Greek	Latin
1	mono	uni
2	duo	bi
3	tri	tri
4	tetra	quad
5	penta	quin
6	hexa	sex
7	hepta	sept
8	octo	oct
9	nona	non
10	deca	dec

These booklets are written and produced by Robert Goodhand

Although the formulae and expressions given have been individually derived and checked errors do creep in. The booklets are also continuously updated.

If you would like the latest issue, just email me at robert.goodhand@gmail.com