Mr. G's little booklet on

Laplace and Fourier

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Mr. G's Little Booklets are

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Laplace Transforms $\mathcal{L}\{f(t)\} = \int_{0-}^{\infty} e^{-st} f(t)\, dt$

Linearity and Constant Theorems

$\mathcal{L}\{A f(t)+B g(t)\} = A \mathcal{L}\{f(t)\} + B \mathcal{L}\{g(t)\}$

Scaling Theorem

$\mathcal{L}\{f(kt)\} = {}^1/_k F({}^s/_k)$

find the transform of $f(t)$ *and then replace* s *by* ${}^s/_k$ *and also divide whole expression by* k

Frequency Shifting Theorem *(exponential damping)*

$\mathcal{L}\{e^{-at} f(t)\} = \mathcal{L}(s + a)$

find the transform of $f(t)$ *and then replace* s *by* $(s + a)$

Derivatives Theorem

$\mathcal{L}\{f'(t)\} = s\mathcal{L}\{f(t)\} - f(0)$

$\mathcal{L}\{f''(t)\} = s^2 \mathcal{L}\{f(t)\} - sf(0) - f'(0)$

$\mathcal{L}\{f^n(t)\} = s^n \mathcal{L}\{f(t)\} - s^{n-1} f(0) - s^{n-2} f'(0) \ldots - f^{n-1}(0)$

Integral Theorem

$\mathcal{L}\{\int_{0-}^{t} f(t)\, dt\} = {}^1/_s \mathcal{L}\{f(t)\}$ $_{0-}$ *indicates capture at* $t = 0$

Frequency Differentiation Theorem

$\mathcal{L}\{t f(t)\} = {}^-F'_s$

If a function is multiplied by t, *then differentiate the Laplace transform wrt* s.

General term is $\mathcal{L}\, t^n f(t) = (^-1)^n F^n(s)\}$ *the* n^{th} *derivative*

$\mathcal{L}\{t e^{-at} f(t)\} = {}^-F'(s + a)$

Frequency Integration Theorem

$\mathcal{L}\{{}^{f(t)}/_t\} = \int_s^{\infty} F(\sigma)\, d\sigma$

Heaviside Unit Step Function

$\mathcal{H}(t) = 0$ for $t < 0$ $\mathcal{H}(t) = 1$ for $t > 0$ $\mathcal{H}(t)$ *can be written* u(t)

$\mathcal{L}\{\mathcal{H}(t)\} = {}^1/_s$

Inverse Transforms

$$\mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st} F(s)\, dt$$

Time Shifting Theorem *(time delay)*

$\mathcal{L}\{\mathcal{H}(t-T)\}$ $\quad e^{-sT}/s$

$\mathcal{L}\{k[\mathcal{H}(t-a)-\mathcal{H}(t-b)]\}$ $\quad k(e^{-as} - e^{-bs})/s$

Shifting any general function by T *multiplies the transform by* e^{-sT}.

$\mathcal{L}\{f(t-T)\,\mathcal{H}(t-T)\}$ $\quad e^{-sT}\mathcal{L}\{f(t)\}$

Dirac's Delta (Impulse) Function

$\delta(t) = 0$ for $t < 0$ $\qquad \delta(t) = 1/\varepsilon$ for $0 < t < \varepsilon$ $\qquad \delta(t) = 0$ for $t > \varepsilon$

$\mathcal{L}\{\delta(t)\} = 1$

$\mathcal{L}\{\delta(t-T)\} = e^{-sT}$

Periodic Function where $f(t) = f(t+T)$

$$\mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt \,/\, (1 - e^{-sT})$$

Differentiation of a Definite Integral

The "repertoire" of Laplace Transfers can be extended by differentiating both sides.

As a general case if $I(t) = \int_a^b f(x,t)\, dx$

Then $\frac{dI}{dt} = \int_a^b f_t(x,t)\, dx$

$= \int_a^b \frac{\partial f}{\partial t}\, dx$

That is the derivative of the integral is the integral of the derivative of the integrand.

So by differentiating the definite integral on the left and the Laplace transform on the right.

we produce new Laplace transforms.

Initial Value Theorem

$f(0^+) = \lim_{s\to\infty} sF(s)$

Final Value Theorem

$f(\infty) = \lim_{s\to 0} sF(s)$ - *all poles in left hand plane*

Notes

The Fourier Transform is the Laplace Transform when s *is purely imaginary* $s = jw$

$\mathcal{L}\{f(t)\}$ *is a linear bijective transform from* f(t) *- time domain to* F(s) *- frequency domain.*

As a consequence differential equations become linear equations capable of direct solution.

Standard Laplace Transforms

$\mathcal{L}\{1\} \times \mathcal{H}(t)$	$= \frac{1}{s}$	*special case* $\mathcal{L}\{t^0\}$
$\mathcal{L}\{k\} \times \mathcal{H}(t)$	$= \frac{k}{s}$	*constant theorem*
$\mathcal{L}\{t\} \times \mathcal{H}(t)$	$= \frac{1}{s^2}$	*ramp*
$\mathcal{L}\{t^n\} \times \mathcal{H}(t)$	$= \frac{n!}{s^{(n+1)}}$	n^{th} *power* $\quad n \in \mathbb{Z}^+$
$\mathcal{L}\{e^{at}\} \times \mathcal{H}(t)$	$= \frac{1}{s-a}$	*exponential attack*
$\mathcal{L}\{e^{-at}\} \times \mathcal{H}(t)$	$= \frac{1}{s+a}$	*exponential decay*
$\mathcal{L}\{1 - e^{-at}\} \times \mathcal{H}(t)$	$= \frac{a}{s(s+a)}$	*exponential approach*
$\mathcal{L}\{e^{-at} - e^{-bt}\} \times \mathcal{H}(t)$	$= \frac{b-a}{(s+a)(s+b)}$	*exponential difference*
$\mathcal{L}\{ae^{-at} - be^{-bt}\} \times \mathcal{H}(t)$	$= \frac{(a-b)s}{(s+a)(s+b}$	*exponential difference*
$\mathcal{L}\{e^{-at}t^n\} \times \mathcal{H}(t)$	$= \frac{n!}{(s+a)^{(n+1)}}$	n^{th} *power with frequency shift*
$\mathcal{L}\{te^{-t}\} \times \mathcal{H}(t)$	$= \frac{1}{(s+1)^2}$	*special case* $a = 1$ *and* $n = 1$
$\mathcal{L}\{te^{-at}\} \times \mathcal{H}(t)$	$= \frac{1}{(s+a)^2}$	*special case* $n = 1$
$\mathcal{L}\{(1 - at)e^{-at}\} \times \mathcal{H}(t)$	$= \frac{s}{(s+a)^2}$	
$\mathcal{L}\{1 - e^{-at} - at\, e^{-at}\} \times \mathcal{H}(t)$	$= \frac{a^2}{s(s+a)^2}$	
$\mathcal{L}\{\cos \omega t\} \times \mathcal{H}(t)$	$= \frac{s}{s^2 + \omega^2}$	*cosine function*
$\mathcal{L}\{\cosh \omega t\} \times \mathcal{H}(t)$	$= \frac{s}{s^2 - \omega^2}$	*hyperbolic cosine function*
$\mathcal{L}\{\sin \omega t\} \times \mathcal{H}(t)$	$= \frac{\omega}{s^2 + \omega^2}$	*sine function*
$\mathcal{L}\{\sinh \omega t\} \times \mathcal{H}(t)$	$= \frac{\omega}{s^2 - \omega^2}$	*hyperbolic sine function*
$\mathcal{L}\{e^{-at} \cos \omega t\} \times \mathcal{H}(t)$	$= \frac{s+a}{(s+a)^2 + \omega^2}$	*exponentially decaying*
$\mathcal{L}\{e^{-at} \cosh \omega t\} \times \mathcal{H}(t)$	$= \frac{s+a}{(s+a)^2 - \omega^2}$	*exponentially decaying*
$\mathcal{L}\{e^{-at} \sin \omega t\} \times \mathcal{H}(t)$	$= \frac{\omega}{(s+a)^2 + \omega^2}$	*exponentially decaying*
$\mathcal{L}\{e^{-at} \sinh \omega t\} \times \mathcal{H}(t)$	$= \frac{\omega}{(s+a)^2 - \omega^2}$	*exponentially decaying*
$\mathcal{L}\{t \cos \omega t\} \times \mathcal{H}(t)$	$= \frac{(s^2 - w^2)}{(s^2 + \omega^2)^2}$	*ramped cosine function*
$\mathcal{L}\{t \cosh \omega t\} \times \mathcal{H}(t)$	$= \frac{-2s\omega^2}{(s^2 - \omega^2)^2}$	*ramped hyperbolic cos. function*
$\mathcal{L}\{t \sin \omega t\} \times \mathcal{H}(t)$	$= \frac{2s\omega}{(s^2 + \omega^2)^2}$	*ramped sine function-resonance*
$\mathcal{L}\{t \sinh \omega t\} \times \mathcal{H}(t)$	$= \frac{-2s\omega}{(s^2 - \omega^2)^2}$	*ramped hyperbolic sin. function*

As these are unilateral Laplace Transforms valid only for $t \geq 0$ *each is* $\times$ *by* $\mathcal{H}(t)$

Further Laplace Transforms

$\mathcal{L}\{ e^{iat} \}$	=	$^{1}/_{s-ia}$	*holds true.*	†
$\mathcal{L}\{ t\, f(t) \}$	=	$^{-d}/_{ds}[\mathcal{L}\{ f(t) \}]$	*differentiate the transform*	
$\mathcal{L}\{ t^{a} f(t) \}$	=	$(-1)^{a}\ ^{d^a}/_{ds^a}[\mathcal{L}\{ f(t) \}]$		
$\mathcal{L}\{ ^{1}/_{2w^2}(\sin\omega t - \omega t\cos\omega t)\}$	=	$^{\omega}/_{(s^2+\omega^2)^2}$	*resonance*	
$\mathcal{L}\{ t^{n} \}$	=	$^{\Gamma(1+n)}/_{s^\wedge(n+1)}$		$n \in P$
$\mathcal{L}\{ \sqrt{x} \}$	=	$^{1}/_{2s}\sqrt{(^{\pi}/_{s})}$		
$\mathcal{L}\{ ^{1}/_{\sqrt{x}} \}$	=	$\sqrt{(^{\pi}/_{s})}$		
$\mathcal{L}\{ \ln t \}$	=	$^{-(\gamma + \ln s)}/_{s}$	γ = *Euler's Constant* 0.57721…	
$\mathcal{L}\{ \ln^2 t \}$	=	$^{\pi^2}/_{6s} + ^{(\gamma+\ln s)^2}/_{s}$		
$\mathcal{L}\{ ^{\sin\omega t}/_{t} \}$	=	$^{-}\tan^{-1}\ ^{w}/_{s}$	*or* $\cot^{-1}\ ^{w}/_{s}$	‡
$\mathcal{L}\{ ^{\sinh\omega t}/_{t} \}$	=	$^{-}\tanh^{-1}\ ^{w}/_{s}$	*or* $^{-}\coth^{-1}\ ^{w}/_{s}$	‡
$\mathcal{L}\{ t^2 \sin\omega t \}$	=	$^{2\omega(3s^2-\omega^2)}/_{(s^2+\omega^2)^3}$		
$\mathcal{L}\{ t^2 \sinh\omega t \}$	=	$^{2\omega(3s^2+\omega^2)}/_{(s^2-\omega^2)^3}$		
$\mathcal{L}\{ t^2 \cos\omega t \}$	=	$^{2(s^3-3\omega^2 s)}/_{(s^2+\omega^2)^3}$		
$\mathcal{L}\{ t^2 \cosh\omega t \}$	=	$^{2(s^3+3\omega^2 s)}/_{(s^2-\omega^2)^3}$		
$\mathcal{L}\{ t^3 \sin\omega t \}$	=	$^{24\omega(s^3-\omega^2 s)}/_{(s^2+\omega^2)^\wedge 4}$		*to be verified*
$\mathcal{L}\{ t^3 \sinh\omega t \}$	=	$^{24\omega(s^3+\omega^2 s)}/_{(s^2-\omega^2)^\wedge 4}$		*to be verified*
$\mathcal{L}\{ t^3 \cos\omega t \}$	=	$^{6(s^\wedge 4-6\omega^2 s^2+w^\wedge 4)}/_{(s^2+\omega^2)^\wedge 4}$		*to be verified*
$\mathcal{L}\{ t^3 \cosh\omega t \}$	=	$^{6(s^\wedge 4+6\omega^2 s^2+w^\wedge 4)}/_{(s^2-\omega^2)^\wedge 4}$		*to be verified*
$\mathcal{L}\{e^{-\xi wnt}\sin[w_n\sqrt{(1-\xi^2)}]t\}$	=	$^{wn\sqrt{(1-\xi^2)}}/_{s^2+2\xi wns+wn^2}$		$\xi < 1$
$\mathcal{L}\{e^{-\xi wnt}\sin[w_n\sqrt{(\xi^2-1)}]t\}$	=	$^{wn\sqrt{(\xi^2-1)}}/_{s^2+2\xi wns+wn^2}$		$\xi > 1$

Notes

As before this is a one sided transform valid for $t > 0$ *so each function is assummed* × *by* $\mathcal{H}(t)$

$\mathcal{L}$\{capacitor\} = $^{1}/_{sC}$ $\mathcal{L}$\{conductor\} = sL

† *Euler's theorem can therefore be used to verify the transforms of trigonometric functions.*

‡ *dividing by t in the t-domain corresponds to integration in the frequency domain*

Fourier Series - Definitions

$$f(x) = \tfrac{1}{2} a_0 + \sum_{n=1}^{\infty} f(x) \left[a_n \cos(nx) + b_n \sin(nx) \right]$$

$$\tfrac{1}{2} a_0 = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x)\, dx$$ *½a_0 is the mean (dc) value*

$$a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \cos(nx)\, dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} f(x) \sin(nx)\, dx$$

Alternatively I could deduce from $a \cos(nx) + b \sin(nx) = c \sin(x+\alpha)$

where $c_1 = \sqrt{(a_1^2+b_1^2)} \ldots c_n = \sqrt{(a_n^2 + b_n^2)}$ *amplitudes*

and $a_n = \tan^{-1}(a_n / b_n)$ *phase angles*

It is better to express odd functions in sine terms and even functions in cosine terms.

Square

$$f(x) = \sin x + \tfrac{1}{3} \sin 3x + \tfrac{1}{5} \sin 5x + \tfrac{1}{7} \sin 7x \ldots$$

$$f(x) = \cos x - \tfrac{1}{3} \cos 3x + \tfrac{1}{5} \cos 5x - \tfrac{1}{7} \cos 7x \ldots$$

Saw Tooth

$$f(x) = \sin x - \tfrac{1}{2} \sin 2x + \tfrac{1}{3} \sin 3x - \tfrac{1}{4} \sin 4x \ldots$$ *($^-$ve ramp)*

$$f(x) = \tfrac{1}{2} \sin 2x + \tfrac{1}{4} \sin 4x + \tfrac{1}{6} \sin 6x \ldots$$ *($^-$ve ramp)*

$$f(x) = \sin x + \tfrac{1}{2} \sin 2x + \tfrac{1}{3} \sin 3x + \tfrac{1}{4} \sin 4x \ldots$$ *($^+$ve ramp)*

$$f(x) = \tfrac{1}{2} \sin 2x - \tfrac{1}{4} \sin 4x + \tfrac{1}{6} \sin 6x \ldots$$ *($^+$ve ramp)*

Triangular

$$f(x) = \sin x - \tfrac{1}{3^2} \sin 3x + \tfrac{1}{5^2} \sin 5x - \tfrac{1}{7^2} \sin 7x \ldots$$

$$f(x) = \cos x + \tfrac{1}{3^2} \cos 3x + \tfrac{1}{5^2} \cos 5x + \tfrac{1}{7^2} \cos 7x \ldots$$

Clipped Waveforms

$$f(x) = \sin x + \tfrac{1}{3^2} \sin 3x + \tfrac{1}{5^2} \sin 5x + \tfrac{1}{7^2} \sin 7x \ldots$$

$$f(x) = \cos x - \tfrac{1}{3^2} \cos 3x + \tfrac{1}{5^2} \cos 5x - \tfrac{1}{7^2} \cos 7x \ldots$$

Notes

Note the sine and cosine terms in "odd" integer waveforms are phase shifted.

Fourier spent one year calculating a_n *and* b_n *terms making at least two fundamental errors along the way and then realised he could have done it in 2 lines. Euler beat him by 30 years.*

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Table of the Error Function

erf(x) occurs in more complex Laplace Transforms

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

as the normal cumulative distribution function $\Phi = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^x e^{-\frac{1}{2}t^2}\, dt$

$$\mathrm{erf}(x) \quad 2\,\Phi(x\sqrt{2}) - 1$$

z	erf(x)	erfc(x)	z	erf(x)	erfc(x)	z	erf(x)	erfc(x)
0.00	0.00000	1.0000	1.00	0.8427	0.1573	2.00	0.9953	0.0047
0.05	0.0564	0.9436	1.05	0.8624	0.1376	2.05	0.9963	0.0037
0.10	0.1125	0.8875	1.10	0.8802	0.1198	2.10	0.9970	0.0030
0.15	0.1680	0.8320	1.15	0.8961	0.1039	2.15	0.9976	0.0024
0.20	0.2227	0.7773	1.20	0.9103	0.0897	2.20	0.9981	0.0019
0.25	0.2763	0.7237	1.25	0.9229	0.0771	2.25	0.9985	0.0015
0.30	0.3286	0.6714	1.30	0.9340	0.0660	2.30	0.9989	0.0011
0.35	0.3794	0.6206	1.35	0.9438	0.0562	2.35	0.9991	0.0009
0.40	0.4284	0.5716	1.40	0.9523	0.0477	2.40	0.9993	0.0007
0.45	0.4755	0.5245	1.45	0.9597	0.0403	2.45	0.9995	0.0005
0.50	0.5205	0.4795	1.50	0.9661	0.0339	2.50	0.9996	0.0004
0.55	0.5633	0.4367	1.55	0.9716	0.0284	2.55	0.9997	0.0003
0.60	0.6039	0.3961	1.60	0.9763	0.0237	2.60	0.9998	0.0002
0.65	0.6420	0.3580	1.65	0.9804	0.0196	2.65	0.9998	0.0002
0.70	0.6778	0.3222	1.70	0.9838	0.0162	2.70	0.9999	0.0001
0.75	0.7112	0.2888	1.75	0.9867	0.0133	2.75	0.9999	0.0001
0.80	0.7421	0.2579	1.80	0.9891	0.0109	2.80	0.9999	0.0001
0.85	0.7707	0.2293	1.85	0.9911	0.0089	2.85	0.9999	0.0001
0.90	0.7969	0.2031	1.90	0.9928	0.0072	2.90	1.0000	0.0000
0.95	0.8209	0.1791	1.95	0.9942	0.0058	2.95	1.0000	0.0000

Greek Alphabet				Principle/Simplest Use	English	Type
alpha	Α	*not used*	α	*first root of quadratic*	a	**a**
beta	Β	*Beta function*	β	*second root of quadratic*	b	**b**
gamma	Γ	*Gamma function*	γ	*Euler's constant*	g	**g**
delta	Δ	*Difference operator*	δ	*small increment*	d	**d**
epsilon	Ε	*not used*	ε	*error*	short e	**e**
zeta	Ζ	*not used*	ζ	*Riemann zeta function*	z	**z**
eta	Η	*not used*	η	*efficiency*	long e	**h**
theta	Θ	*asymp. tight bound*	θ	*angle*	th	**q**
iota	Ι	*not used*	ι	*imaginary unit*	i	**i**
kappa	Κ	*not used*	κ	*curvature*	k	**k**
lambda	Λ	*diag. matrix eigen-values*	λ	*failure rate*	l	**l**
mu	Μ	*not used*	μ	*population mean*	m	**m**
nu	Ν	*not used*	ν	*poisson ratio*	n	**n**
xi	Ξ	*grand canonical ensemble*	ξ	*damping coefficient*	x	**x**
omicron	Ο	*limiting behaviour function*	ο	*generally not used*	short o	**o**
pi	Π	*Product operator*	π	*ratio* $^c/_d$ *circle*	p	**p**
rho	Ρ	*not used*	ρ	*correlation coefficient*	r	**r**
sigma	Σ	*summation*	σ	*standard deviation*	s	**s**
tau	Τ	*not used*	τ	*mean lifetime*	t	**t**
upsilon	Υ	*Bessel function*	υ	*generally not used*	u	**u**
phi	Φ	*cumulative function*	ϕ	*golden ratio*	ph	**f**
phi (alt.)	ϑ	*not used*	φ	**normal function** **scalar potential**	ph	**j**
chi	Χ	*probability function*	χ^2	*chi-squared prob.function*	ch	**c**
psi	Ψ	*not used*	ψ	*wave function*	ps	**y**
omega	Ω	*mathematical constant*	ω	*angular frequency*	long o	**w**
stigma	ς					**v**
pomega			ϖ	*angular velocity*		**v**

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Orders of Magnitude

septillionth	yocto-	y	10^{-24}	septillion	yotta-	Y	10^{24}
sextillionth	zepto-	z	10^{-21}	sextillion	zetta-	Z	10^{21}
quintillionth	atto-	a	10^{-18}	quintillion	exa-	E	10^{18}
quadrillionth	femto-	f	10^{-15}	quadrillion	peta-	P	10^{15}
trillionth	pico-	p	10^{-12}	trillion	tera-	T	10^{12}
billionth	nano-	n	10^{-9}	billion	giga-	G	10^{9}
millionth	micro-	μ	10^{-6}	million	mega-	M	10^{6}
thousandth	milli-	m	10^{-3}	thousand	kilo-	k	10^{3}
hundredth	centi-	c	10^{-2}	hundred	hecto-	h	10^{2}
tenth	deci-	d	10^{-1}	ten	deca-	da	10^{1}
one	-	-	10^{0}	one	-	-	10^{0}

Mathematical Constants - 30 decimals (last place not rounded)

pi	π	=	*3.14159 26535 89793 23846 26433 83279…*
exponential	e	=	*2.71828 18284 59045 23536 02874 71352…*
Pythagoras's	$\sqrt{2}$	=	*1.41421 35623 73095 04880 16887 24209…*
	$\sqrt{3}$	=	*1.73205 08075 68877 29352 74463 41505…*
	$\log 2$	=	*0.69314 71805 59945 30941 72321 21458…*
golden ratio	ϕ	=	*1.61803 39887 49894 84820 45868 34365…*
Euler-Mascheroni	γ	=	*0.57721 56649 01532 86060 65120 90082…*
Feigenbaum's	δ	=	*4.66920 16091 02990 67185 32038 20466…*
	$\xi(2)$	=	*1.64493 40668 48226 43647 24151 66646…*
Apery's	$\xi(3)$	=	*1.20205 69031 59594 28539 97381 61511…*
	$\xi(4)$	=	*1.08232 32337 11138 19151 60036 96541…*
Euler's	$\xi(5)$	=	*1.03692 77551 43369 92633 13654 86457…*
	$\xi(6)$	=	*1.01734 30619 84449 13971 45179 29790…*
	e^{π}	=	*23.14069 26327 79269 00572 90863 67948…*

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Prime Numbers (in columns of 25)

2	101	233	383	547	701	877	1049	1229	1429	1597	1783
3	103	239	389	557	709	881	1051	1231	1433	1601	1787
5	107	241	397	563	719	883	1061	1237	1439	1607	1789
7	109	251	401	569	727	887	1063	1249	1447	1609	1801
11	113	257	409	571	733	907	1069	1259	1451	1613	1811
13	127	263	419	577	739	911	1087	1277	1453	1619	1823
17	131	269	421	587	743	919	1091	1279	1459	1621	1831
19	137	271	431	593	751	929	1093	1283	1471	1627	1847
23	139	277	433	599	757	937	1097	1289	1481	1637	1861
29	149	281	439	601	761	941	1103	1291	1483	1657	1867
31	151	283	443	607	769	947	1109	1297	1487	1663	1871
37	157	293	449	613	773	953	1117	1301	1489	1667	1873
41	163	307	457	617	787	967	1123	1303	1493	1669	1877
43	167	311	461	619	797	971	1129	1307	1499	1693	1879
47	173	313	463	631	809	977	1151	1319	1511	1697	1889
53	179	317	467	641	811	983	1153	1321	1523	1699	1901
59	181	331	479	643	821	991	1163	1327	1531	1709	1907
61	191	337	487	647	823	991	1171	1361	1543	1721	1913
67	193	347	491	653	827	1009	1181	1367	1549	1723	1831
71	197	349	499	659	829	1013	1187	1373	1553	1733	1933
73	199	353	503	661	839	1019	1193	1381	1559	1741	1949
79	211	359	509	673	853	1021	1201	1399	1567	1747	1951
83	223	367	521	677	857	1031	1213	1409	1571	1753	1973
89	227	373	523	683	859	1033	1217	1423	1579	1759	1979
97	229	379	541	691	863	1039	1223	1427	1583	1777	1999

Notes

Prime Number Theorem states that the number of primes up to n, $\pi_n \sim {}^{n}/_{\ln(n)}$

Alternatively the n^{th} *prime number* $p_n \sim n \ln(n)$. *So* $p_{300} \sim 300 \ln 300 = 1711$ *(cf* 1999*)*

If $\mathrm{li} = \int {}^{dt}/_{\ln t}$ *then* $\mathrm{Li}(x) = {}_2\int^x {}^{dt}/_{\ln t} = \mathrm{li}(x) - \mathrm{li}(2)$ *is a better approximation to* $\pi(x)$

Goodhand's conjecture states the percent proportion of primes approximately equals the percent that ${}^{n}/_{\ln(n)}$ *underestimates p(n). Hence* $\pi(n)$ *better* $\approx \frac{1}{2}(1 - \sqrt{\{1 - {}^{4}/_{\ln(n)}\}}$

Counting

No.	***Greek***	***Latin***
1	*mono*	*uni*
2	*duo*	*bi*
3	*tri*	*tri*
4	*tetra*	*quad*
5	*penta*	*quin*
6	*hexa*	*sex*
7	*hepta*	*sept*
8	*octo*	*oct*
9	*nona*	*non*
10	*deca*	*dec*

These booklets are written and produced by Robert Goodhand

Although the formulae and expressions given have been individually derived and checked errors do creep in. The booklets are also continuously updated.

If you would like the latest issue, just email me at robert.goodhand@gmail.com