Mr. G's little booklet on

Number Patterns

Issue 5.0 16/16

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Determining the n^{th} term from a forward difference table

Suppose you are given the following sequence and have to determine the n^{th} term

5	17	47	101	185

Produce the forward difference table

12	30	54	84
18	24	30	
6	6		

It took three rows to produce a common difference so the sequence must be a cubic.

The cubic term is the common difference divided by $3\times2\times1$ *ie* $6\div6 = 1$

So now we know it's $1n^3 + ?n^2 + ?n + ?$

Now we deduct this cubic term from the original sequence.

	1	2	3	4	5
	5	17	47	101	185
minus	1	8	27	64	125
New sequence is	4	9	20	37	60

Now we repeat the exercise

5	11	17	23
6	6	6	

It took two rows this time to produce a common difference so it must be a quadratic.

The quadratic term is the common difference divided by 2×1 *ie* $6\div2 = 3$

So now we know it's $n^3 + 3n^2 + ?n + ?$

Now we deduct this quadratic term from our new sequence and repeat the exercise.

	1	2	3	4	5
	4	9	20	37	60
minus	3	12	27	48	75
New sequence is	1	-3	-7	-11	-15
	-4	-4	-4	-4	

It took just one row this time to produce a common difference so the new sequence is linear

The linear term is the common difference divided by 1 *ie* $^-4\div1 = {}^-4$

So now we know $1n^3 + 3n^2 - 4n + ?$

Finally we just step back to the 0^{th} *term to find the constant* 5

So now we know the n^{th} *term is* $1n^3 + 3n^2 - 4n + 5$

Worked Example - Determining the nth term

Dividing up regions in a circle from n ponts on the circumference

0	1	2	3	4	5	6	7
1	1	2	4	8	16	31	57
	0	1	2	4	8	15	26
	1	1	2	4	7	11	
		0	1	2	3	4	
		1	1	1			

It took four rows to produce a common difference so the sequence must be a quartic.

The quartic term is the common difference divided by 4×3×2×1 ie 1÷24

So now we know it's $\,^{1}/_{24}\, n^4 + ?n^3 + n^2 + ?n + 1$

Now we deduct this cubic term from the original sequence.

0	1	2	3	4	5	6	7
1	1	2	4	8	16	31	57
0.00	0.04	0.67	3.38	10.67	26.04	54.00	100.04
1.00	0.96	1.33	0.63	-2.67	-10.04	-23.00	-43.04
	-0.04	0.38	-0.71	-3.29	-7.38	-12.96	-20.04
	0.42	-1.08	-2.58	-4.08	-5.58	-7.08	
		-1.50	-1.50	-1.50	-1.50	-1.50	

The cubic term is the common difference divided by 3×2×1 *ie* ⁻1.5÷6 = ⁻1÷4

So now we know it's $\,^{1}/_{24}\, n^4 - \,^{1}/_{4}n^3 + n^2 + ?n + 1$

Now we deduct this cubic term from our new sequence and repeat the exercise.

0	1	2	3	4	5	6	7
1.00	0.96	1.33	0.63	-2.67	-10.04	-23.00	-43.04
0.00	-0.25	-2.00	-6.75	-16.00	-31.25	-54.00	-85.75
1.00	1.21	3.33	7.38	13.33	21.21	31.00	42.71
	0.21	2.13	4.04	5.96	7.88	9.79	11.71
	1.92	1.92	1.92	1.92	1.92	1.92	

The quadratic term is the common difference divided by 2×1 *ie* 1.92 ÷ 2 = $^{23}/_{24}$

So now we know it's $\,^{1}/_{24}\, n^4 - \,^{1}/_{4}n^3 + \,^{23}/_{24}\, n^2 + ?n + 1$

Now we deduct this quadratic term from our new sequence and repeat the exercise.

0	1	2	3	4	5	6	7
1.00	1.21	3.33	7.38	13.33	21.21	31.00	42.71
0.00	0.96	3.83	8.63	15.33	23.96	34.50	46.96
1.00	0.25	-0.50	-1.25	-2.00	-2.75	-3.50	-4.25
	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75	-0.75

The linear term is the common difference divided by 1 *ie* $^{-3}/_{4}$

So now we know it's $\,^{1}/_{24}\, n^4 - \,^{1}/_{4}n^3 + \,^{23}/_{24}\, n^2 - \,^{3}/_{4}n + 1$

Any number to any power as a sum of gnomic numbers

Case A : Powers of "Odd" Numbers

								n^{x-1}												n^x
						1	+	3	+	5							=	9	=	3^2
				1	+	3	+	5	+	7	+	9					=	25	=	5^2
		1	+	3	+	5	+	7	+	9	+	11	+	13			=	49	=	7^2
1		3		5		7		9		11		13		15		17	=	81	=	9^2
						7	+	9	+	11							=	27	=	3^3
				21	+	23	+	25	+	27	+	29					=	125	=	5^3
		43	+	45	+	47	+	49	+	51	+	53	+	55			=	343	=	7^3
73		75		77		79		81		83		85		87		89	=	729	=	9^3
						25	+	27	+	29							=	81	=	3^4
				121	+	123	+	125	+	127	+	129					=	625	=	5^4
		337	+	339	+	341	+	343	+	345	+	347	+	349			=	2401	=	7^4
721		723		725		727		729		731		733		735		737	=	6561	=	9^4

Case B : Powers of "Even" Numbers

						$n^{x-1}-1$		$n^{x-1}+1$										
						1	+	3							=	4	=	2^2
				1	+	3	+	5	+	7					=	16	=	4^2
		1	+	3	+	5	+	7	+	9	+	11			=	36	=	6^2
1		3		5		7		9		11		13		15	=	64	=	8^2
						3	+	5							=	8	=	2^3
				13	+	15	+	17	+	19					=	64	=	4^3
		31	+	33	+	35	+	37	+	39	+	41			=	216	=	6^3
57		59		61		63		65		67		69		71	=	512	=	8^3
						7	+	9							=	16	=	2^4
				61	+	63	+	65	+	67					=	256	=	4^4
		211	+	213	+	215	+	217	+	219	+	221			=	1296	=	6^4
505		507		509		511		513		515		517		519	=	4096	=	8^4

Powers from Series

Start each block by writing successive integers in pairs, triplets etc.

Each successive line has one fewer integer in the block

Each digit is the sum of the integer above and the integer to the left.

Each pattern ends with an integer that is 1,2,3… raised to the next power. Neat!

1	2		3	4		5	6		7	8		9	10	
1	=	1^2	4	=	2^2	9	=	3^2	16	=	4^2	25	=	5^2

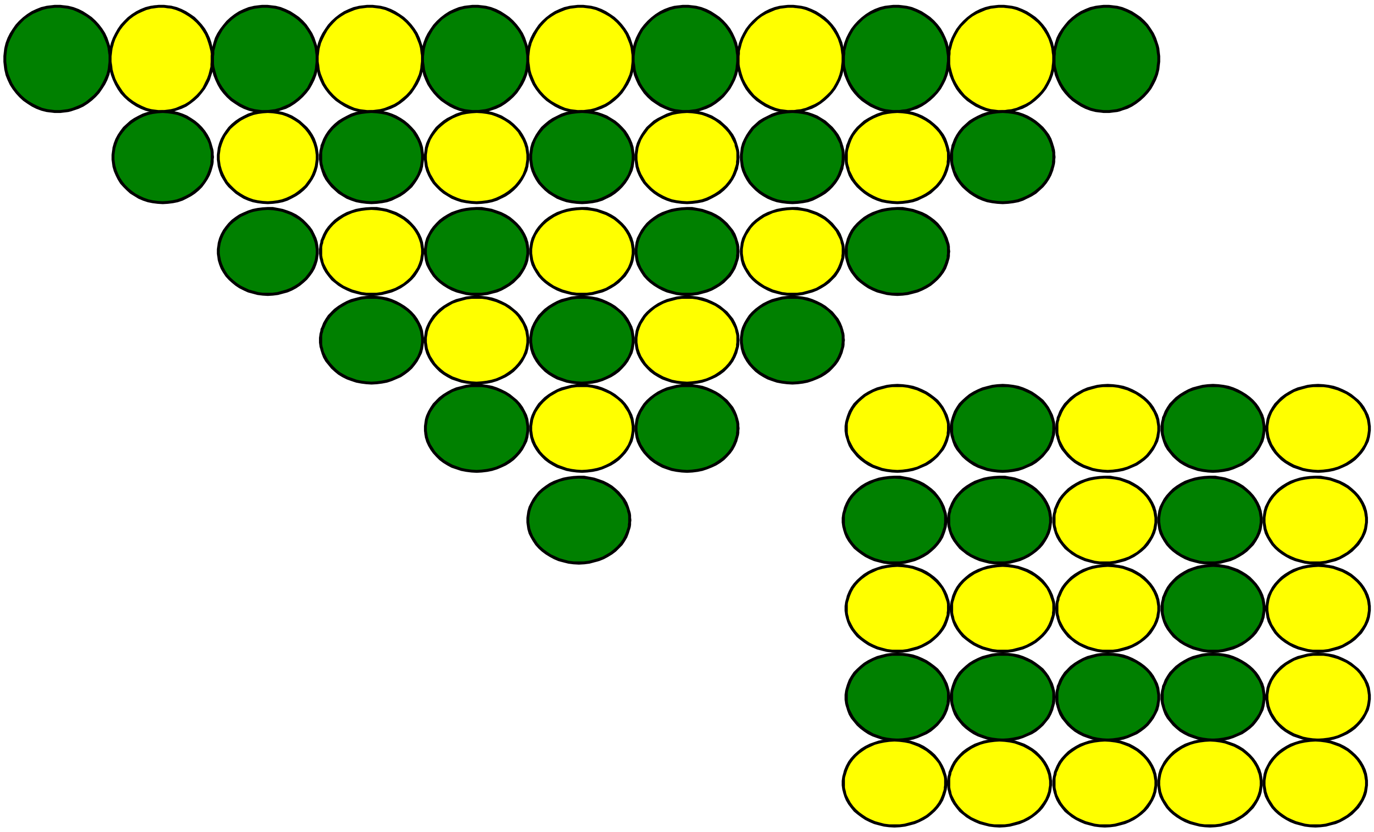
1	2	3	4	5	6	7	8	9	10	11	12
1	3		7	12		19	27		37	48	
1	=	1^3	8	=	2^3	27	=	3^3	64	=	4^3

1	2	3	4	5	6	7	8	9	10	11	12
1	3	6		11	17	24		33	43	54	
1	4			15	32			65	108		
1	=	1^4		16	=	2^4		81	=	3^4	

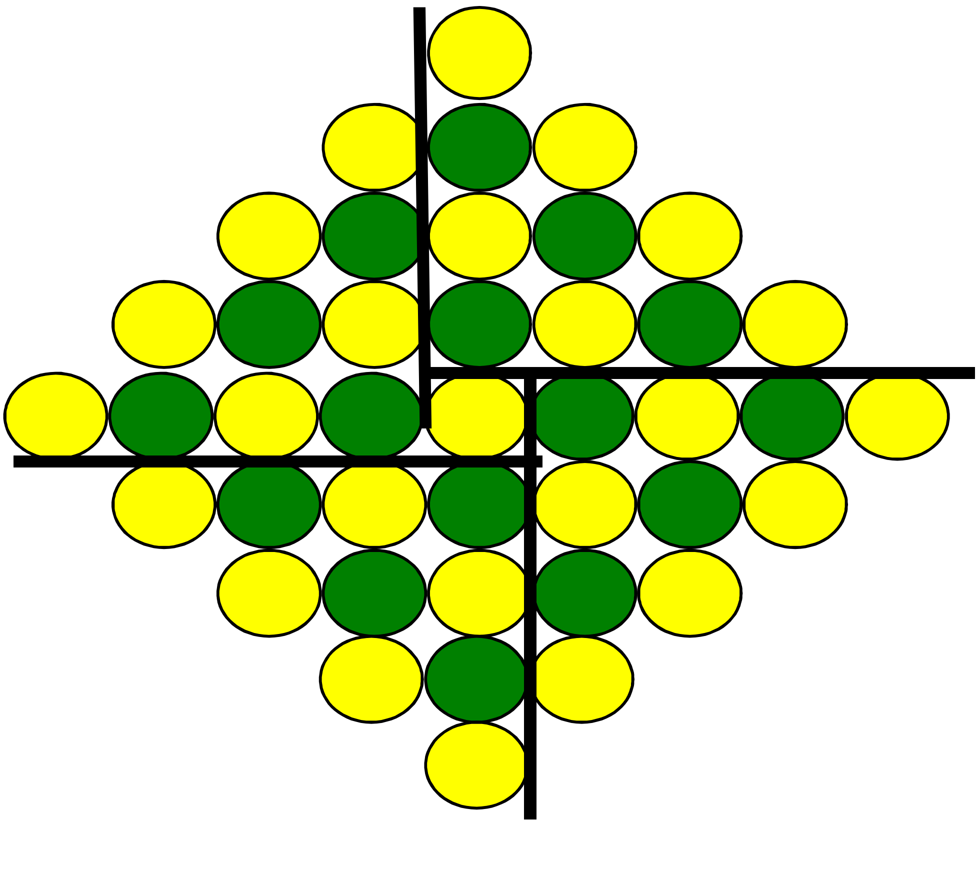
1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	3	6	10		16	23	31	40		51	63	76	90
1	4	10			26	49	80			131	194	270	
1	5				31	80				211	405		
1	=	1^5			32	=	2^5			243	=	3^5	

1	2	3	4	5	6	7	8	9	10	11	12
1	3	6	10	15		22	30	39	49	60	
1	4	10	20			42	72	111	160		
1	5	15				57	192	240			
1	6					63	192				
1	=	1^6				64	=	2^6			

Square Numbers

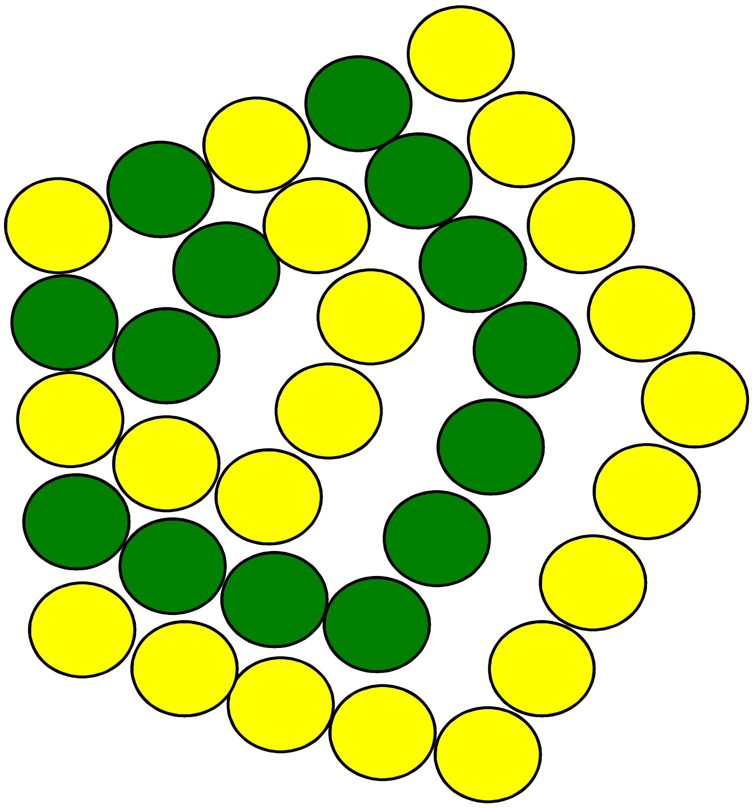


Centred Square Numbers

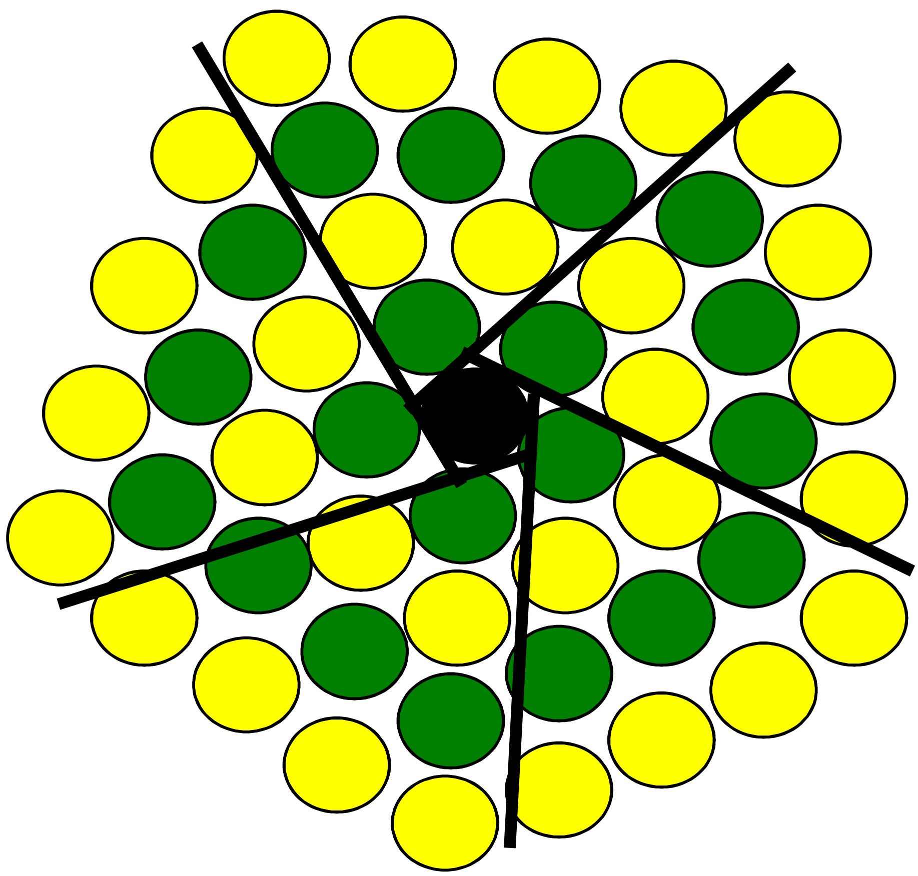


Centred Square = 4 × Triangular + 1

Pentagonal Numbers

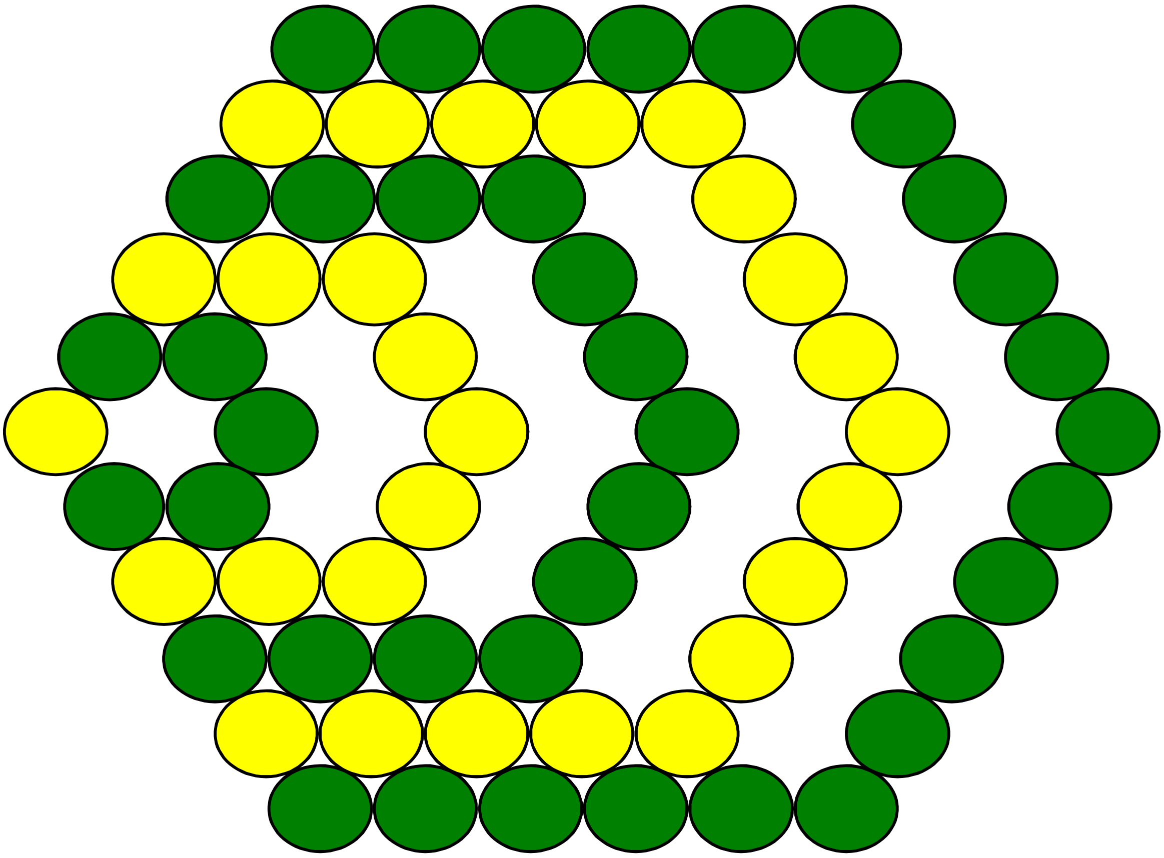


Centred Pentagonal Numbers

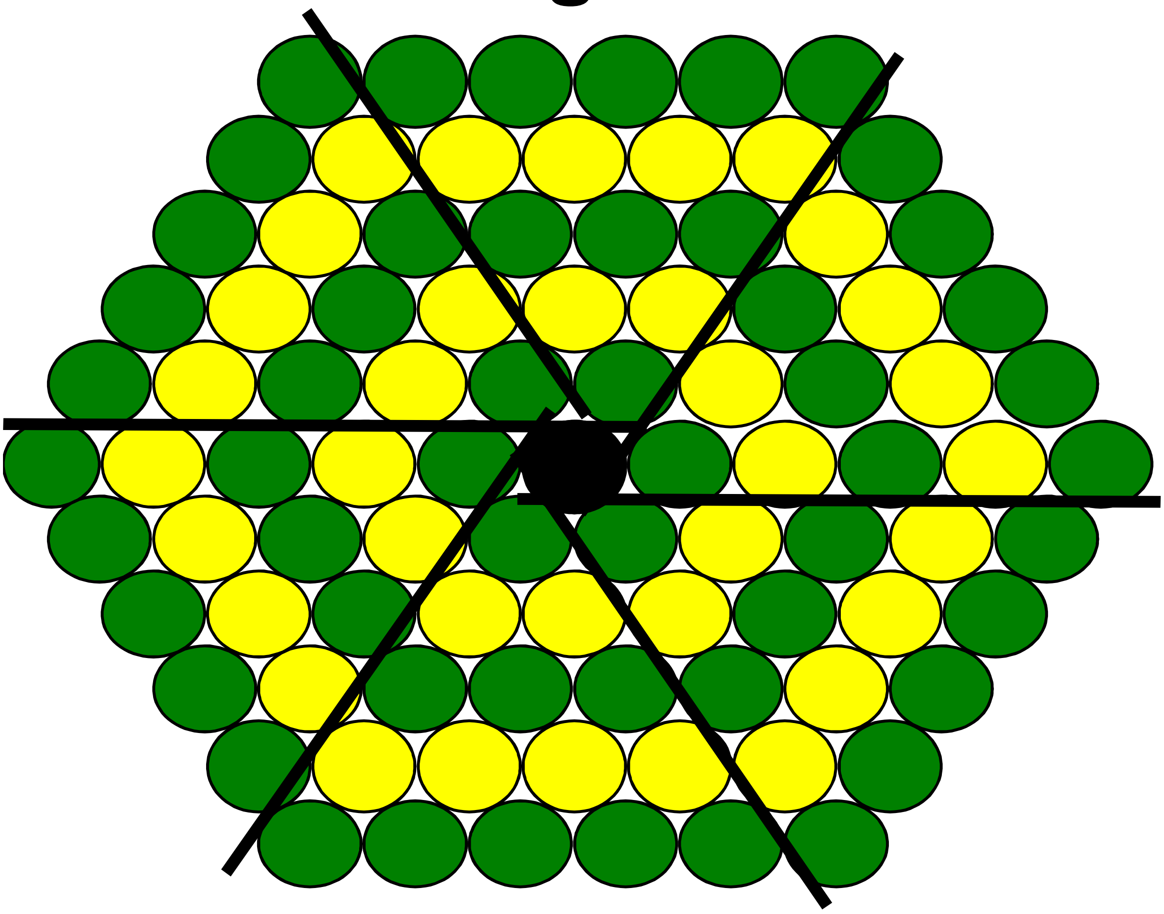


Centred Pentagonal = 5 × Triangular + 1

Hexagonal Numbers



Centred Hexagonal Numbers



Centred Hexagonal = 6 × Triangular + 1

Summary of Key Number Sequences

Cardinal

Cardinal		1+1+1+1…					Σ repunits			
1	2	3	4	5	6	7	8	9	10	»

Polygon Numbers (1st order)

Triangular		1+2+3+4…			Δ_n		Σ cardinals			
1	3	6	10	15	21	28	36	45	55	»
Square		1+3+5+7…			$\Delta_n + 1 \times \Delta_{n-1}$ or Σ gnomonic					
1	4	9	16	25	36	49	64	81	100	»
Pentagonal		1+4+7+10…			$\Delta_n + 2 \times \Delta_{n-1}$					
1	5	12	22	35	51	70	92	117	145	
Hexagonal		1+5+9+13…			$\Delta_n + 3 \times \Delta_{n-1}$			*(cf odd Δ nos.)*		
1	6	15	28	45	66	91	120	153	190	

Pyramidal Numbers (2nd order)

Σ polygon numbers gives

Tetrahedral *(Triangular Pyramidal)* (∇) †										
1	4	10	20	35	56	84	120	165	220	»
Square Pyramidal (sum squares)							$\nabla_n + 1 \times \nabla_{n-1}$			
1	5	14	30	55	91	140	204	285	385	»
Pentagonal Pyramidal							$\nabla_n + 2 \times \nabla_{n-1}$			
1	6	18	40	75	126	196	288	405	550	
Hexagonal Pyramidal							$\nabla_n + 3 \times \nabla_{n-1}$			
1	7	22	50	95	161	252	372	525	715	

4D Polygonal Nos. (3rd order)

Σ pyramidal numbers gives

Pentatopes *(3rd diagonal of Pascal's Triangle)*					Σ tetrahedral					
1	5	15	35	70	126	210	330	495	715	»
4D Square Pyramidal					Σ pyramidal					
1	6	20	50	105	196	336	540	825	1210	»
4D Pentagonal Pyramidal					Σ pentagonal pyramidal					
1	7	25	65	140	266	462	750	1155	1705	
4D Hexagonal Pyramidal					Σ hexagonal pyramidal					
1	8	30	80	175	336	588	960	1485	2200	

† *These are the number presents for successive days of "12 days of Christmas". 12th day is 364*

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Gnomonic (odd)

Σ 2 consecutive cardinals

1	3	5	7	9	11	13	15	17	19

Centred Polygonal Numbers

Σ n Δ numbers + 1

Centred Triangular

Σ 3 Δ numbers + 1

1	4	10	19	31	46	64	85	109	136

Centred Square

Σ 4 Δ numbers + 1 or 2 cons. *f* nos.

1	5	13	25	41	61	85	113	145	181

Centred Pentagonal

Σ 5 Δ numbers + 1

1	6	16	31	51	76	106	141	181	226

Centred Hexagonal

Σ 6 Δ numbers + 1

1	7	19	37	61	91	127	169	217	271

3D Centred Polygonal Numbers

Σ centred polygonal numbers

"Follow on" Triangular

Σ centred Δ or 3 x ∇

1	5	15	34	65	111	175	260	369	505

Octahedral †

Σ centred or 2 cons. pyramidal

1	6	19	44	85	146	231	344	489	670

3D Centred Pentagonal

Σ centred pentagonal

1	7	23	54	105	181	287	428	609	835

Cubic or Hexahedral †

Σ centred hexagonal

1	8	27	64	125	216	343	512	729	1000

4D Centred Polygonal Numbers

Σ 3D centred polygonal nos.

Doubly Triangular

Σ "follow on" Δ or 3 x pentatopes

1	6	21	55	120	231	406	666	1035	1540

4D Centred Square

Σ octahedral numbers or 2 x 4Δ pyramidal

1	7	26	70	155	301	532	876	1365	2035

4D Centred Pentagonal

Σ 3Δ centred pentagonal

1	8	31	85	190	371	658	1086	1695	2530

Square Triangular *(sum of cubes)*

Σ cubic or Δ^2

1	9	36	100	225	441	784	1296	2025	3025

†*Platonic Solids*

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Sequences and Sums - find the next sequence and sum

1) Cardinal Numbers

$$1 + 2 = 3$$
$$4 + 5 + 6 = 7 + 8$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15$$
$$16 + 17 + 18 + 19 + 20 = 21 + 22 + 23 + 24$$

What comes next?

2) "Odd" and Square Numbers

$1 = 1^2$	$S_1 = 1$
$1 + 3 = 2^2$	$S_2 = 4$
$1 + 3 + 5 = 3^2$	$S_3 = 9$
$1 + 3 + 5 + 7 = 4^2$	$S_4 = 16$

What comes next?

3) "Odd" and Cube Numbers

$1 = 1^3$	$C_1 = 1$
$3 + 5 = 2^3$	$C_2 = 8$
$7 + 9 + 11 = 3^3$	$C_3 = 27$
$13 + 15 + 17 + 19 = 4^3$	$C_4 = 64$

What comes next?

4) Triangular Numbers

$1 = 1$	$T_1 = 1$
$1 + 2 = 3$	$T_2 = 3$
$1 + 2 + 3 = 6$	$T_3 = 6$
$1 + 2 + 3 + 4 = 10$	$T_4 = 10$

What comes next?

Sequences and Sums - find the next sequence and sum

5) Square Numbers Pattern

$$3^2 + 4^2 = 5^2$$

$$10^2 + 11^2 + 12^2 = 13^2 + 14^2$$

$$21^2 + 22^2 + 23^2 + 24^2 = 25^2 + 26^2 + 27^2$$

$$36^2 + 37^2 + 38^2 + 39^2 + 40^2 = 41^2 + 42^2 + 43^2 + 44^2$$

What comes next?

6) Triangular and Square Numbers

$$0 + 1 = 1^2 \qquad S_1 = 1$$

$$1 + 3 = 2^2 \qquad S_2 = 4$$

$$3 + 6 = 3^2 \qquad S_3 = 9$$

$$6 + 10 = 4^2 \qquad S_4 = 16$$

What comes next?

7) Triangular Numbers Pattern

$$1 + 3 + 6 = 10$$

$$15 + 21 + 28 + 36 = 45 + 55$$

$$66 + 78 + 91 + 105 + 120 = 136 + 153 + 171$$

What comes next?

8) Cubes and Triangular Numbers

$$1^3 = 1^2 \qquad (T_1)^2$$

$$1^3 + 2^3 = 3^2 \qquad (T_2)^2$$

$$1^3 + 2^3 + 3^3 = 6^2 \qquad (T_3)^2$$

$$1^3 + 2^3 + 3^3 + 4^3 = 10^2 \qquad (T_4)^2$$

What comes next?

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Sequences and Sums - find the next sequence and sum

9) Triangular Numbers and Quartic Powers

$2^4 = 1 + 15 \quad (T_1 + T_5)$

$3^4 = 15 + 66 \quad (T_5 + T_{11})$

$4^4 = 66 + 190 \quad (T_{11} + T_{19})$

$5^4 = 190 + 435 \quad (T_{19} + T_{29})$

What comes next?

10) Centred Hexagonal Numbers

$1 = 1 \quad H_{c1} = 1$

$1 + 6 = 7 \quad H_{c2} = 7$

$1 + 6 + 12 = 19 \quad H_{c3} = 19$

$1 + 6 + 12 + 18 = 37 \quad H_{c4} = 37$

What comes next?

11) Centred Hexagonal Numbers and Cube Numbers

$1 = 1^3 \quad C_1 = 1$

$1 + 7 = 2^3 \quad C_2 = 8$

$1 + 7 + 19 = 3^3 \quad C_3 = 27$

$1 + 7 + 19 + 37 = 4^3 \quad C_4 = 64$

What comes next?

12) Pythagorean Triples

as $4 + 5 = 3^2$ then $3^2 + 4^2 = 5^2$

as $12 + 13 = 5^2$ then $5^2 + 12^2 = 13^2$

as $24 + 25 = 7^2$ then $7^2 + 24^2 = 25^2$

as $40 + 41 = 9^2$ then $9^2 + 40^2 = 41^2$

What comes next?

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13) Cubes and Differences Triangular Numbers

$2^3 = 3^2 - 1^2 \quad (T_2 - T_1)$

$3^3 = 6^2 - 3^2 \quad (T_3 - T_2)$

$4^3 = 10^2 - 6^2 \quad (T_4 - T_3)$

$5^3 = 15^2 - 10^2 \quad (T_5 - T_4)$

What comes next?

14) Sum Cubes and Product Squares

$1^3 = \frac{1}{4} \times 1^2 \times 2^2$

$1^3 + 2^3 = \frac{1}{4} \times 2^2 \times 3^2$

$1^3 + 2^3 + 3^3 = \frac{1}{4} \times 3^2 \times 4^2$

$1^3 + 2^3 + 3^3 + 4^3 = \frac{1}{4} \times 4^2 \times 5^2$

What comes next?

15) Centred Triangular Numbers

$3 \times 1 + 1 = 4 \quad T_{c2}$

$3 \times 3 + 1 = 10 \quad T_{c3}$

$3 \times 6 + 1 = 19 \quad T_{c4}$

$3 \times 10 + 1 = 31 \quad T_{c5}$

What comes next?

16) Perfect Numbers (sum factors = 2n)

$1 + 2 + 3 = 6$

$1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$

Double last number sequence and add 1. If prime then sum series is perfect number.

7x2+1 = 15. Not prime. 15 x 2 + 1 = 31. Prime.

$1 + 2 + 3 + 4 + 5 + \dots + 31 = 496$

31x2+1 = 63. Not prime. 63 x 2 + 1 = 127. Prime.

$1 + 2 + 3 + 4 + 5 + \dots + 127 = 8128$

What comes next? (This is tricky)

Sequences and Sums - find the next sequence and sum

17) Centred Square Numbers

$4 \times 1 + 1 = 5$ $\quad S_{c2}$

$4 \times 3 + 1 = 13$ $\quad S_{c3}$

$4 \times 6 + 1 = 25$ $\quad S_{c4}$

$4 \times 10 + 1 = 41$ $\quad S_{c5}$

What comes next? What other "4 type" sequence gives this?

18) Centred Pentagonal Numbers

$5 \times 1 + 1 = 6$ $\quad P_{c2}$

$5 \times 3 + 1 = 16$ $\quad P_{c3}$

$5 \times 6 + 1 = 31$ $\quad P_{c4}$

$5 \times 10 + 1 = 51$ $\quad P_{c5}$

What comes next? What other "5 type" sequence gives this?

19) Centred Hexagonal Numbers

$6 \times 1 + 1 = 7$ $\quad H_{c2}$

$6 \times 3 + 1 = 19$ $\quad H_{c3}$

$6 \times 6 + 1 = 37$ $\quad H_{c4}$

$6 \times 10 + 1 = 61$ $\quad H_{c5}$

What comes next?

20) "Odd" number sequences summing to quartics.

$1 = 1^4$

$7 + 9 = 2^4$

$25 + 27 + 29 = 3^4$

$61 + 63 + 65 + 67 = 4^4$

$121 + 123 + 125 + 127 + 129 = 5^4$

What comes next? Look back at examples 2) and 3)

What is the median number in each sequence?

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Answers to Sequences and Sums

1) $25 + 26 + 27 + 28 + 29 + 30 = 31 + 32 + 33 + 34 + 35$

2) $1 + 3 + 5 + 7 + 9 = 5^2$

3) $21 + 23 + 25 + 27 + 29 = 5^3$

4) $1 + 2 + 3 + 4 + 5 = 15$

5) $55^2 + 56^2 + 57^2 + 58^2 + 59^2 + 60^2 = 61^2 + 62^2 + 63^2 + 64^2 + 65^2$

6) $10 + 15 = 5^2$

7) $190 + 210 + 231 + 253 + 276 + 300 = 325 + 351 + 378 + 406$

8) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 15^2$

9) $6^4 = 861 + 435$ $(T_{29} + T_{41})$

10) $1 + 6 + 12 + 18 + 24 = 61$ H_4

11) $1 + 7 + 19 + 37 + 61 = 5^3$

12) $11^2 + 60^2 = 61^2$

13) $6^3 = 21^2 - 15^2$

14) $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = \frac{1}{4} \times 5^2 \times 6^2$

15) $3 \times 15 + 1 = 46$

16) $1 + 2 + 3 + 4 + \ldots + 8191 = 33550336$

17) $4 \times 15 + 1 = 61$

18) $5 \times 15 + 1 = 76$

19) $6 \times 15 + 1 = 91$

20) $211 + 213 + 215 + 217 + 219 + 221 = 1296$

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N ~ omial Distributions

Trinomial Distribution *A027907*

1 1 1 *Each number is the sum of the one above and two to the left*

1 2 3 2 1 *terms of expansion* $(1 + x + x^2)^n$

1 3 6 7 6 3 1

1 4 10 16 19 16 10 4 1

1 5 15 30 45 51 45 30 15 5 1

eg $(1 + x + x^2)^3 = 1 + 3x + 6x + 7x + 6x + 3x + x$

Quadrinomial Distribution *A008287*

1 1 1 1 *Each number is the sum of the one above and three to the left*

1 2 3 4 3 2 1 *terms of expansion* $(1 + x + x^2 + x^3)^n$

1 3 6 10 12 12 10 6 3 1

1 4 10 20 31 40 44 40 31 20 10 4 1

1 5 15 35 65 101 135 155 155 135 101 65 35 15 5 1

eg $(1 + x + x^2 + x^3)^3 = 1 + 3^x + 6x^2 + 10x^3 + 12x^4 + 12x^5 + 10x^6 + 6x^7 + 3x^8 + x^9$

Quintinomial Distribution *A035343*

1 1 1 1 1 *Each number is the sum of the one above and four to the left*

1 2 3 4 5 4 3 2 1 *terms of expansion* $(1 + x + x^2 + x^3 + x^4)^n$

1 3 6 10 15 18 19 18 15 10 6 3 1

1 4 10 20 35 52 68 80 85 80 68 52 35 20 10 4 1

1 5 15 35 70 121 185 255 320 365 381 365 320 255 185 121 70 35 15 5 1

Sextinomial Distribution *A166322 A063260*

1 1 1 1 1 1 *Each number is the sum of the one above and five to the left*

1 2 3 4 5 6 5 4 3 2 1 *terms of expansion* $(1 + x + x^2 + x^3 + x^4 + x^5)^n$

1 3 6 10 15 21 25 27 27 25 21 15 10 6 3 1

1 4 10 20 35 56 80 104 125 140 146 140 125 104 80 56 35 20 10 4 1

1 5 15 35 70 126 205 305 420 540 651 735 780 780 735 651 540 420 305 205 126 *etc.*

these are the number of ways of totalling any given sum of n dice thrown together

Notes

For n dice, s sides, the coefficient for total k is $\sum_{i=0}^{\lfloor (k-n)/s \rfloor} (-1)^i \binom{n}{i} \binom{k-si-1}{n-1}$

For $n = 4$, $s = 6$ *and a total* $k = 13$ *this reduces to* ${}^{12}C_3 - 4 \times {}^{6}C_3 = 140$

I discovered the sextinomial coefficients when examining all the combinations of three dice.

Later I expanded that to n~omial coefficients. Finally I constructed the triangles.

Then I discovered Euler had beaten me to it by 250 years though the layout may be original.

As n *and* $s \rightarrow \infty$ *we get the normal (Gaussian) distribution.*

rg

Coding a Coordinate Pair into a Single Number

Let $z = \frac{1}{2}\{ (a + b)^2 + 3a + b \}$

b \ a	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
0	0	2	5	9	14	20	27	35	44	54	65	77	90	104	119
1	1	4	8	13	19	26	34	43	53	64	76	89	103	118	134
2	3	7	12	18	25	33	42	52	63	75	88	102	117	133	150
3	6	11	17	24	32	41	51	62	74	87	101	116	132	149	167
4	10	16	23	31	40	50	61	73	86	100	115	131	148	166	185
5	15	22	30	39	49	60	72	85	99	114	130	147	165	184	204
6	21	29	38	48	59	71	84	98	113	129	146	164	183	203	224
7	28	37	47	58	70	83	97	112	128	145	163	182	202	223	245
8	36	46	57	69	82	96	111	127	144	162	181	201	222	244	267
9	45	56	68	81	95	110	126	143	161	180	200	221	243	266	290
10	55	67	80	94	109	125	142	160	179	199	220	242	265	289	314
11	66	79	93	108	124	141	159	178	198	219	241	264	288	313	339
12	78	92	107	123	140	158	177	197	218	240	263	287	312	338	365

Notes

Using the above function any pair of numbers can be converted into a unique integer.

But how would you convert a given integer into a unique coordinate pair?

An algorithm would be to determine maximum n such that $T_n < z$

Then $b = z - T_n$ *and* $a = n - b$.

Extend the problem into three/four or more coordinate pairs

Further Observations

The "0" column is the triangular numbers

The "1" column are termed central polygonal numbers. Investigate these.

Find a general formula for subsequent columns.

Investigate the horizontal sequences

rg

Counting

No.	***Greek***	***Latin***
1	*mono*	*uni*
2	*duo*	*bi*
3	*tri*	*tri*
4	*tetra*	*quad*
5	*penta*	*quin*
6	*hexa*	*sex*
7	*hepta*	*sept*
8	*octo*	*oct*
9	*nona*	*non*
10	*deca*	*dec*

These booklets are written and produced by Robert Goodhand

Although the formulae and expressions given have been individually derived and checked errors do creep in. The booklets are also continuously updated.

If you would like the latest issue, just email me at robert.goodhand@gmail.com

rg