Rules of Propositional Calculus (after Hofstadter)

There are 6 symbols used in propositional calculus

∧ **AND**
∨ **OR**
⩒ **exclusive OR (XOR)**
⇒ **implication**
⇔ **equivalence**
¬ **negation**

The symbols **p** and **q** stand in place of well-formed propositions.
A "well formed proposition" or combination of propositions and symbols that are themselves well formed in accordance with the rules below are termed "theorems" if they are always true..
Ask for the "bookmark" summarising their truth tables.

Joining Rule

If **p** and **q** are theorems then **p ∧ q** is also a theorem.

Separation Rule

If **p ∧ q** is a theorem then both **p** and **q** are theorems.

Double Tilde Rule

¬ ¬ can be deleted from any theorem.

Deduction Theorem (Fantasy Rule)

If **q** can be derived when **p** is assumed to be a theorem then **p ⇒ q** is a theorem.

Contrapositive Rule

p ⇒ q and **¬q ⇒ ¬p** are interchangeable

De Morgan's Rules

¬p ∧ ¬q and **¬(q ∨ p)** are interchangeable.
¬p ∨ ¬q and **¬(q ∧ p)** are interchangeable.

Transposition Rule 1 (the Switcheroo rule)

p ⇒ q and **¬p ∨ q** are interchangeable.

Transposition Rule 2

p ⇔ q and **¬(p ⩒ q)** are interchangeable.

Note : We don't actually need either of the symbols ⇒ or ⇔ because we can substitute other symbols but using them makes life a lot easier.
It's even more tricky to eliminate the **⩒** symbol and only a purist would insist on replacing **⩒** with **(p ∧ ¬q) v (¬p ∧ q)** every time it needed to be used.
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