Mr G's Little Booklet on

Bell's Inequality

$$E(2\theta) \leq 2E(\theta)$$

Bell's inequality is derived by assuming local causality when measuring pairs of photons generated and subsequently passing through polarisers aligned at an angle. In actual experiments the inequality is violated.

$E(\theta)$ is the error rate – the proportion of pairs that don't correlate.

Bell's Inequality discounts any possibility of a sub-quantum theory of hidden variables to explain the correlation that might restore local causality.

The conclusion is that the world is not locally causal.

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Why does every website make such a meal of explaining Bell's inequality while professing to be the idiot's guide? All are to my view incomprehensible, using terms that require detailed background reading. So here's the real idiot's guide.

Start with light – once thought to be a continuous electromagnetic wave but now broken down into "packets" called photons. A beam of light can be plane polarised – that is the e part of the e-m wave is aligned in one direction (with the m part orthogonal) – let's say vertically. If you pass an unpolarised beam of light through a polariser – that is all the individual photons with different orientations - half the photons will get through and half will be blocked. So what determines which and how many photons get through?

If you consider light a continuous beam the calculation is easy. Treat the light beam as a vector with intensity and direction and split that into two orthogonal vectors, one lined up with the polariser and one at right angles. The proportion of the beam of light that gets through is just $\cos^2\theta$ and the proportion blocked is $\sin^2\theta$ and $\cos^2\theta + \sin^2\theta = 1$.

However as the beam is made up of individual photons, each photon must "decide" what to do so the terms now become probabilities. So a photon aligned at 10° has a 97% chance making it through and a photon aligned at 80° has a 3% chance of getting through. You might have thought those aligned less than 45° make it and those aligned greater don't but it's not quite that simple. The outcome however is the same.

To demonstrate this we integrate $\cos^2\theta$ over the range 0° to 90° and get

$$_{0^\circ}\!\!\int^{90^\circ} \cos^2\theta \, d\theta) = {}_{0^\circ}\!\!\int^{90^\circ} ½ \,(1+ \cos2\theta) \, d\theta)$$

$$= [½\theta + ¼\sin2\theta]^{90}_{0} = 45°$$

and we divide that by the range 90° to get the proportion photons passing through as ½.

Now assume a process where photons are created in pairs each with the same orientation and travelling in opposite directions to 2 aligned polarisers. We don't know what the actual orientation of the pair is but each hits its respective polariser. Quantum Mechanics will tell you they have no orientation until measured – whatever!

The weird thing is that if we have a click detector beyond the polarisers they will either both click or both stay silent within the accuracy of the experiment. If the polarisers are unequally spaced and the first clicks then for a period of time even though we don't know the actual orientation we know what will happen when the second photon reaches its polariser.

Now to a Quantum Mechanist knowing something before its measured is itself a bit of a "no no". So the photons appear to be engaged in some personal conspiracy either by determining beforehand their collective fates or communicating in flight. Before they set off they each seemed to know what they would do when reaching some arbitrary polariser. On the face of it either they are communicating while in flight or they have within them some agreed mechanism – a "look up" table that tells them what to do for each angle encountered. Yet that undermines our original assumption that the decision is made at the instant the photon encounters the polariser.

It gets worse because simple statistics demonstrates there can be no hidden variable that performs such a role. Here's why.

If you first align your polarisers then the correlation will be 100%. Move one polariser an angle A (for simplicity A < 45°) anticlockwise. Now you'll get a lesser proportion of matched clicks. Let's say on A occasions of a run of 100 we get a click at one polariser and no click at the other. Then return the first polariser to the vertical and rotate the second at an angle B. Now on B occasions we get click/no click. Now rotate the first polariser A anticlockwise and the second B clockwise and run the experiment again. You should expect there to be fewer click/no clicks than A + B because a double failure will count as a success. If we equate A and B put succinctly we have

$$\cos^2 2\theta < 2\cos^2\theta - 1$$

which is certainly true for $\theta < 45°$.

Let P(A) and P(B) be the respective probabilities of passage of one photon through the polariser

The probability of a match – two clicks or two no clicks – is

$$P(A) \times P(B) \text{ (both make it)}$$

plus

$$\{ 1 - P(A) \} \times \{ 1 - P(B) \}$$

(neither make it)

and this extra term ensures that the overall number of matchings – both click or both don't - should be more than the sum of the individual matches of the previous two experiments – *whatever the "look-up" table might be that determines* P(A) and P(B).

However QM predicts that the failure for a match is given by $1 - \cos^2\theta$ where θ is the total angle between the polarisers – ie $\theta = A + B$

Go back to the vector calculation. You'll predict more click/no click occasions and that's exactly what happens when you perform the experiment – well so I'm told.

That is the proportion of failures is given by $1 - \cos^2 (A + B)$

$$1 - \cos^2 (A + B) > (1 - \cos^2 A) + (1 - \cos^2 B)$$

When rearranged gives

$\cos^2 (A + B) < \cos^2 A + \cos^2 B - 1$

Set A =20 and B= 30 for example

$$0.41 < 0.88 + 0.75 - 1$$

The only time the inequality fails is when we set A and B = 45° when we have 100% failure on each side because the polarisers are aligned at 90º.

Now there are four ways out of this quandary

1) Accept the Copehhagen interpretation – there are no models of reality that explain this result
2) Accept they somehow there is instantaneous communication. This is termed superluminal transfer of negentropy.
3) Accept contrafactual definitiveness fails – could things have been otherwise?
   a) If contrafactualness fails we have superdeterminism
   b) If definiteness fails we have "many worlds"

Take your pick!