Curvature

Definitions

Let there be a curve $y = f(x)$

At a point on the curve let the arc length be s and the angle of the tangent at that point θ.

Define the "first curvature" as κ.

Let the radius of curvature be τ.

We define

$\kappa = {}^{d\theta}/_{ds}$

and $\tau = \left| {}^{ds}/_{d\theta} \right|$ *absolute value*

Hence $\tau = {}^{1}/_{\kappa}$

Derivation of κ

${}^{dy}/_{dx} = \tan\theta$

${}^{d^2y}/_{dx^2} = \sec^2\theta \times {}^{d\theta}/_{dx}$

${}^{d\theta}/_{dx} = {}^{d^2y}/_{dx^2} \cos^2\theta$

Now $\cos\theta = {}^{dx}/_{ds}$

so ${}^{d\theta}/_{ds} = {}^{d\theta}/_{dx} \times {}^{dx}/_{ds}$

$= {}^{d^2y}/_{dx^2} \cos^2\theta \times \cos\theta$

$= {}^{d^2y}/_{dx^2} / \sec^3\theta$

Hence $\kappa = {}^{d^2y}/_{dx^2} / \sqrt{(1 + \tan^2\theta)^3}$

and $\tau = \{ 1 + ({}^{dy}/_{dx})^2 \}^{3/2} / {}^{d^2y}/_{dx^2}$

Summary to date

I now need to read up a bit more about this. Wikipedia is usually great chunks of pure maths copied across for the benefit of "who knows who" and Wolfram while claiming to pitch maths at everyone seems to define "everyone" as degree level and above.

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