Differentiating Trig Functions

Circular Functions

$^{d}/_{dx} \sin(m\theta + \phi) = m \cos(m\theta + \phi)$

Now consider $\sin(\theta + \tfrac{1}{2}\pi)$

$\sin(\theta + \tfrac{1}{2}\pi)$

$= \sin\theta \cos\tfrac{1}{2}\pi + \cos\theta \sin\tfrac{1}{2}\pi$

$= 0 + \cos\theta$

Therefore $m \cos(m\theta + \phi)$

$= m \sin(m\theta + \phi + \tfrac{1}{2}\pi)$

and

$^{d}/_{dx} \sin(m\theta + \phi) = m \sin(m\theta + \phi + \tfrac{1}{2}\pi)$

So differentiating a circular function phase shifts the angle by $\tfrac{1}{2}\pi$ and introduces the multiplier m.

If we say $f_0 = \sin(m\theta + \phi)$ and represent successive differentiations by f_1 f_2 f_3 etc. then $f_n = m^n \sin(m\theta + \phi + \tfrac{1}{2}n\pi)$

and if $g_0 = \cos(m\theta + \phi)$

then $g_n = m^n \cos(m\theta + \phi + \tfrac{1}{2}n\pi)$

Hence this demonstrates exactly why differentiation of a circular function cycles after $n = 4$ because $4 \times \tfrac{1}{2}\pi = 360°$

Specifically

$^{d}/_{dx} \sin\theta = \cos\theta$

$^{d2}/_{dx2} \sin\theta = -\sin\theta$

$^{d3}/_{dx3} \sin\theta = -\cos\theta$

$^{d4}/_{dx4} \sin\theta = \sin\theta$

Hyperbolic Functions

$^{d}/_{dx} \sinh(ax + b) = a \cosh(ax + b)$

Now consider $\sinh(a + \tfrac{1}{2}\pi i)$

$\sinh(a + \tfrac{1}{2}\pi i)$

$= \sinh a \cosh\tfrac{1}{2}\pi i + \cosh a \sinh\tfrac{1}{2}\pi i$

$= 0 + i \cosh a$

because $\cosh ix = \cos x$ and $\cos\tfrac{1}{2}\pi = 0$

and $\sinh ix = i \sin x$ and $i\sin\tfrac{1}{2}\pi = i$

Therefore $a \cosh(ax + b)$

$= ai \sinh(ax + b + \tfrac{1}{2}\pi i)$ and

$^{d}/_{dx} \sinh(ax + b) = ia \sinh(ax + b + \tfrac{1}{2}\pi i)$

So differentiating a hyperbolic function phase shifts the hyperbolic angle by $\tfrac{1}{2}\pi i$ and introduces the multiplier ia.

Similarly we will discover

$\sinh(a + i\pi) = -\sinh a$ and

$\cosh(a + i\pi) = -\cosh a$

Thus the periodicity of sinh and cosh is still 2π because in addition we need $i^4 = 1$ Alternatively you add two lots of $i\pi$ to cancel out the negative.

Neverthless conveniently we now have

$^{d}/_{dx} \sinh x = \cosh x$

$^{d}/_{dx} \cosh x = \sinh x$

because sinh and cosh don't have the asymmetry of sin and cos. ∞ *rg*