Teaching Division of Fractions

Introduction

I used to explain fractions like this to sets you have to convince $^3/_4 + {}^2/_3$ doesn't equal $^5/_7$. Suppose I got one merit in Maths, two in English and three in science how many merits do I have in total? Six. But suppose I came 1st in Maths, 2nd in English and 3rd in Science where would I be overall? Certainly not sixth. Probably top the students might say though we don't actually have enough information to decide exactly.

Then I'd tell them I was letting them into a big secret. There are actually two types of numbers. The top numbers of the fraction are our usual cardinal numbers , the ones that count. The bottom numbers are "ordinal" numbers – the ones that order – and you can never mix them up.

The problem with this explanation is that it's complete nonsense. Later on when you have to calculate $^2/_3 \div {}^4/_5$ you give them the craziest instruction. Change the ÷ to a × and swap over the 4 and 5 in the second fraction to $^5/_4$. So $^2/_3 \div {}^4/_5 = {}^2/_3 \times {}^5/_4 = {}^5/_6$.
It makes no sense and flies in the face of all that careful groundwork you've laid.

New Approach

Then I hit upon this. You are an aid worker somewhere feeding children. You get 200 chapatis delivered with the instruction "4 chapatis feed one child". So how many children can you feed? $200 \div 4 = 50$

Next week you get 25 giant chapatis delivered and the instructions say "half a chapati feeds one child". Now common sense says you can still feed 50 children and we have $25 \div ½ = 50 \Rightarrow 25 \times {}^2/_1 = 50$

Although this seems admirably sensible the teacher might still have the awkward feeling he /she doesn't actually know exactly why this works especially when you move onto say $^8/_5 \div {}^5/_4 \Rightarrow {}^8/_5 \times {}^4/_5 = {}^{32}/_{25}$.
Well your bottom set might not get it but try this $^8/_5 \div {}^5/_4 = (^8/_5) / (^5/_4)$
So we clearly have a numerator on the top and a denominator on the bottom and we have a rule – whatever you do to the top you do to the bottom.
So multiply both by $^4/_5$.
$(^8/_5) / (^5/_4) \Rightarrow (^8/_5 \times {}^4/_5) / (^5/_4 \times {}^4/_5)$
The denominator now reduces to one and we're left with $(^8/_5) / (^5/_4) \Rightarrow (^8/_5 \times {}^4/_5) = {}^{32}/_{25}$

Easy.

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