There are 3 Cosine Double Angle Formulae but only one Sine? Why?

We have

$\cos 2\theta = 1 - 2\sin^2\theta$

$\quad = 2\sin^2\theta - 1$

$\quad = \cos^2\theta - \sin^2\theta$

but only

$\sin 2\theta = = \cos^2\theta - \sin^2\theta$

$\sin 2\theta = = - 2 \sin\theta \ \cos\theta$

This is a specific of the power series where we can express

$\cos^n$ in cosines (n odd or even)

$\sin^n$ in cosines (n even)

$\sin^n$ in sines (n odd)

but **not**

$\cos^n$ in sines (n odd or even)

$\sin^n$ in sines (n even)

$\sin^n$ in cosines (n odd)

The same asymmetry is reflected in all hyperbolic relationships. It also surfaces in the seemingly innocuous

if $\quad \sinh x = \tan y$

$\quad \cosh x = \sec y$

$\quad \tanh x = \sin y$

Why does the cosh invert?

Because we already know

$\quad \cosh x = \cos ix$

and the introduction of i gives the clue to the asymmetry.

We have to move everything into the complex plane to see the full picture

Circular functions have period 2π

Hyperbolic functions have period $2i\pi$

In establishing our sin cos relationships we allocate sine to the IMAGINARY (the y) axis.

Now $e^{ix} = \cos x + i \sin x$

let $x \quad = 2A$

$e^{i2A} = \cos 2A + i \sin 2A$

but $e^{i2A} = e^{iA} \times e^{iA}$

$\quad = (\cos A + i \sin A)^2$

Thus we multiply out and equate real and imaginary parts to reveal

$\cos 2A = \cos^2 A - \sin^2 A$

which we manipulate to produce the other two but

$\text{Sin}2A = 2 \sin A \cos A$

which we're stuck with.

As we have $\quad \cos ix = \cos x$

$\quad \sinh ix = i \sin x$

and whenever we square up the i sin function the i disappears to give us – sin which is Osborne's rule.

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