Mr G's Little Booklet on

Eddington's Equation

$10m^2 - 136m + 1 = 0$

as detailed in The Expanding Universe and Fundamental Theory

Written by Robert Goodhand

Preface

Arthur Eddington does not use consistent nomenclature between The Expanding Universe (EU) printed in 1940 and Fundamental Theory (FT) posthumously published in 1948.

For example in EU, N refers to the number of protons in the universe while in FT N refers to the number of protons and electrons. Even Eddington himself seems unsure on occasions which was which though in the early days of the development of his theory a factor of 2 was immaterial.

EU uses G for the gravitational constant but switches to k in FT to avoid confusion with the tensor G. I however need k's in abundance for various factors in EU so I use G throughout.

Therefore I have used a hybrid consistent nomenclature throughout, supplementing with suffixes to avoid confusion. Readers should be aware of this if keen enough to cross-reference any equation.

At one point I use α and β as the roots of a quadratic and not to be confused with α representing the fine structure constant nor β the bond factor used elsewhere.

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Review of "The Expanding Universe" and "Fundamental Theory"

Introduction

Back in the 50's when sons asked their dads those really difficult questions that would get them all embarrassed they'd cop out and give them a little book and say "read that son". It was no different in my case and eventually I too got my little book. It was The Expanding Universe (EU). I first read it in about 1960 when I was 10. I was immediately fascinated that there could be a link between the very small and the very large evidenced by the equation

$$R / \sqrt{N} = e^2 / mc^2$$

where R is the radius of curvature of empty space (strictly R_s), N the number of protons in the universe (strictly N_p), e electron charge, m mass (strictly m_e) and c the velocity of light. I tried to recreate the calculations for many years without success until first the availability of the electronic calculator and then the spreadsheet gave me renewed enthusiasm.

However the next hurdle was the cgs system of units. EU told me e^2 / mc^2 had the dimensions of length which seemed to imply charge was not fundamental. That required an extensive study into the cgs system of units largely replaced by SI from 1970.

I tackled the project on and off for 50 years but the final result was immensely disappointing. The roots of the quadratic equation with later refinements

$$10m^2 - 136m^{\sqrt{N}}/_R + N / R^2 = 0$$

where m is now an arbitrary variable are supposedly the masses of the proton and electron in "natural" units adjusted by the scale factor $\sqrt{N} / R$.

A worksheet quickly demonstrates that the two adjusting values Eddington obtains in EU 135.9264 (δ_1) and 0.0073569 (δ_2) are in fact independent of any chosen values N and R and also Hubble's "constant" H_0. Yet he justifies his hypothesis by noting the closeness of δ_1 to the original assumption of 136. However the value 136 can be varied widely and the calculated constant δ_1 will always lie with 0.1% of whatever value is chosen for the fine structure constant.

The coincidence is that the roots of the basic equation $10m^2 - 136m + 1 = 0$ are in the ratio 1847.6 which just happens to lie close to the ratio

$$m_p / m_e = 1847.$$

The ratio is highly dependent on the chosen values 10 and 136 and any slight deviation from these values shifts the ratio significantly. Consequently that coincidence determines the inevitable outcome of subsequent equations whatever other values are chosen.

Having spent 30 years searching bookshops for a copy of Fundamental Theory (FT) I only had to be patient while Amazon rose to prominence and a copy was painlessly delivered to me by the postman – the first anti-climax. The second was that FT was completely impenetrable to me and I gave up on the project for 10 years or so.

However on retirement I made a renewed effort. Using a worksheet I managed to link together into one complete coherent set of equations the basics of FT.

Eddington takes three measured values – the speed of light, Faraday's constant and Rydberg's constant to establish the system of units. He then throws in a precise value for N_T and α - the fine structure constant - to determine with high accuracy theoretical values for m_p and m_e.

However despite my continued immense admiration of the man, I'm still left with the distinct feeling that there was a bit of sleight of hand somewhere.

Never-the-less Eddington's commitment to the cosmological constant, now central to modern cosmology, may yet prove to be his final legacy.

Robert Goodhand

A man with too much time on his hands

The Expanding Universe

After a fascinating insight into the wonders of an expanding universe and 4 dimensional space, the mathematical adventure starts on page 54 when Eddington gives me 5 values

a) speed of recession

H_0 ≈ 528 km/sec/megaparsec distance

b) Initial Einstein radius of universe

R_0' ≈ 328 megaparsecs

R_0'' ≈ 1 068 million light years

c) Total mass of Einstein universe

M_e = 2.14E+55 gm

≈ 1.08E+22 sun masses.

d) Number protons in universe

N_p = 1.29E+79

and Eddington assumes equal to the number electrons.

e) Initial mean density

ρ_e = 1.05E–27 gm/cm^3

= 1 hydrogen atom per 1580 cm^3.

f) Cosmological constant

λ = 9.8E–55 cm^{-2}

together with the fascinating statement *"these results are inter–related; when one of them is known all the others can be deduced accurately".*

So my first task was to demonstrate this and discover what other assumptions I needed to make along the way. First I had to get to grips with all the units.

The astronomical unit is the average distance from the earth to the sun and as Eddington is only working to three significant figures, I might as well use the modern definition - approx.

1.50E+13 cm. (k_1).

The megaparsec is defined as 648000π au's (k_2) so I multiply k_1 by k_2 to get k_3, that is 3.09E+24 cm. in a megaparsec.

To complete preliminaries I need the speed of light c. Eddington used 2.99776E+10 cm so I multiply that by seconds in a year, averaging for leap years every 4 years to get a light year

c' = 9.46E+17 cm.

Thus k_4, light years in a megaparsec is k_3 / c' or 3.26E+6, that is a little over 3 in a parsec.

So keeping track of all the units I determined the relative speed of recession

$v_0 = H_0 \times 10^5 / k_3$.

Now Eddington doesn't tell me at this point that the radius of curvature of empty space

$R_s \quad = c / v_0$

which he calculates as 1.75E+27

but he does tell me that the initial radius $\quad R_e \quad = R_s / \sqrt{3}$

so I have the initial Einstein radius of the universe $R_0 \quad = 1.01E+27$ cm

and dividing this by k_3 get

$R_0' \quad = 328$ megaparsecs

my first "result".

I divide this by k_4 to get my second result $R_0'' \quad = 1068$ million light years.

Eddington is more helpful with the key equation from Einstein

$GM / c^2 = \frac{1}{2} \pi R_e$

and from this I get the Einstein mass of the universe $M_e \quad = \frac{1}{2} \pi R_e c^2 / G$

= 2.14E+55 grams my third result.

I divide this by the modern estimate of the sun's mass,

$M_s \quad \approx 1.989E+33$ grams and get

$M_e' \quad \approx 1.08E+22$ sun's masses.

Even today I read there are about a 100 thousand million stars in a galaxy and 100 thousand million galaxies in the universe which is 10^{22} stars in total These are impressive calculations.

At this point it appears Eddington assumes the mass of the proton

$m_p \quad = 1.66E–24$ grams

so given a negligible electron mass he deduces the number of protons in the Universe as $N_p \quad = M_e / m_p$

= 1.29E+79 my fifth result.

The rest follows readily.

The volume of spherical space

$V_e \quad = 2 \pi^2 R_s^3 = 2.04E+82$ cm^3

Hence the initial mean density of the Einstein universe is given by

$\rho_e \quad = M_e / V_e \ = 1.05E+27$ gm / cm^3.

Alternatively V_e / N_p gives me one hydrogen atom per 1580 cm^3.

The only remaining calculation is the cosmological constant conveniently derived from

$\lambda \quad = 1 / R_0^2$ or $= 9.8E–55$ cm^{-2}

That disposes of the basic algebra to derive these initial results - all hanging on Hubble's constant of

$H_o \quad = 528$ km / sec / megaparsec.

Now at page 84, I am given two key equations.

From wave mechanics he postulates

$R_s / \sqrt{N_p} = e^2 / mc^2$

with some debate that there might be a further numerical factor k_n involved.

and from relativity theory we have

$N_p / R_s = \pi / 2\sqrt{3} \times c^2 / Gm_p$

Multiplying left and right to eliminate R_s

I get $\sqrt{N_p} \quad = e^2\pi / m_e.2\sqrt{3}Gm_p$

giving a new result $N_p \ = 4.44E+78$

which is of the right order – remember I still have an unknown factor k_n.

If I were to assume $k_n \ = \sqrt{3}$

for no better reason than then $\sqrt{3}$ crops up frequently in other Einstein equations then $N_p \quad = 1.33E+79$

which is fairly close to his first-cut of

$N_p \quad = 1.29E+79$

Now matters get deep. Eddington is searching for a quadratic equation whose roots give the masses of proton and electron and that the coefficients of that equation are predetermined rather than just random. He never explains philosophically why this should be.

Let's start with $am^2 + bm + c = 0$

with roots α and β.

Recalling all that mathematics on roots of equations that I never thought would have any practical use

$\alpha + \beta \quad = {}^{-b}/_{a}$

$\alpha\beta \quad = {}^{c}/_{a}$

${}^{\alpha\beta}/_{\alpha+\beta} \quad = {}^{-c}/_{b}$

Now Eddington starts with a basic

$$10m^2 - 136m + 1$$

Reading Eddington's "Space Time and Gravitation" I discover on page 143 that the "10" derives from the ten potentials obtained when slicing up a 4 dimensional mesh – 4+3+2+1 or the 4th triangular number. There is however no specific justification why it might be inserted here other than a casual mention at the bottom of page 82. "136" reflects Eddington's belief that the fine structure constant was exactly 137 modified by the packing density ${}^{137}/_{136}$.

The bond factor raised to various powers will feature prominently in Fundamental Theory (FT).

This equation, in whatever units, gives

$m_p / m_e =$

$(136+\sqrt{18456}) \ / \ (136 - \sqrt{18456})$

$= 1847.6$

which is pretty close to the m_p / m_e measured ratio of 1847. So however I tinker with this equation I must not lose sight of this.

If I modify the equation with some arbitrary constant k_a I get

$$10m^2 - 136k_a m + k_a^2$$

but I still get m_p / m_e =

$$(136+\sqrt{18456}) / (136 - \sqrt{18456})$$

with the k_a factor conveniently cancelling out.

This now gives Eddington considerable latitude because he can now fix the "b" coefficient to be $m_p + m_e$
and when adjusting with "b" he then simply inserts the square into c to retain the correct ratio.
By reasoning that is not fully explained he lets $k_a = \sqrt{N_p} / R$
to produce an answer in natural units without clarifying how these natural units relate to say "grams".
Nevertheless he now produces
$10m^2 - 136m\sqrt{N_p} / R_s - N_p / R_s^2$
which I already know will now automatically give the correct ratio and value for m_h , the mass of the hydrogen atom.

Solving this equation using available values for R_s and N_p gives values
m_p = 2.79E+13 and
m_e = 1.51E+10
in some undefined "natural" units.

Dividing these numbers back into the value for $\sqrt{N} / R_s$ does indeed give the final equations on page 89
$135.9264\ m_e = \sqrt{N_p} / R_s$ and
$0.073569\ m_p = \sqrt{N_p} / R_s$
where 135.9264 is my δ_1 term and 0.073569 is my δ_2 term.

He proudly justifies the thought processes by observing 135.9264 is close to the original assumption of 136.

Regrettably there is a flaw in the final argument that never seems to have been commented on before.

I could choose any reasonable value for the fine structure constant and working through the same mathematics, the error will always be under 0.1% because other terms dominate.

Fundamental Theory

In Appendix D, I summarise all the numerous relationships, many involving various manifestations of the bond factor β.

The central quadratic has metamorphosed into

$$10m^2 - 136m_o m + \beta^{5/6} m_o^2$$

Now the comparison particle m_0 which is introduced as a convenient adjusting term is defined as $m_0 \quad = 10m_h / 136$.

It has to be this value because our "b" coefficient is the sum of the roots

$\alpha = m_p$ and $\beta = m_e$ with $\alpha + \beta = m_h$ the mass of the hydrogen atom.

No previous commentator appears to have taken issue with the arbitrary introduction of m_0.

The adjusted bond factor at the end just sneaks the ratio m_p / m_e down a bit to agree the better with the latest measured value available to him.

The central equation to FT is

$m_0 \quad = \frac{3}{4} \beta^{1/6} \hbar \sqrt{(4/5 N)} / cR_0$ *(page 81)*

and m_0 defines m_h as above.

Further $h \quad = 137e^2 / c$

"137" being the fine structure constant

I can also derive m_h directly from

$m_h \quad = 2M_0 / N$ where

$M_0 \quad = \pi R_0 c^2 / 2G$ and

$R_0 \quad = 2\sqrt{N}\sigma$

As in the Expanding Universe, Eddington focuses on two expressions,

$R_0 / N_p \quad = Gm_h / \pi c^2$

from wave mechanics and

$R_o / \sqrt{N_p} = {}^{136}/_{10} \, ({}^{9}/_{20}) \, \beta h / 2\pi c m_h$

from relativity

Thus I now have three methods to recover the value of m_h and each time I get $m_h \quad = 1.67368E-24$ grams

In summary, Eddington takes three measured constants

$c \quad = 2.99776E+10$ cm/sec

From this, he again derives the light year $c_y \quad = 9.46021E+27$ cm.

He quotes Ryberg's constant

$R \quad = 1.09678E+5 \text{ cm}^{-1}$

and Faraday's constant as adjusted

$F' \quad = 9.57356E+3$ abcoulombs

He then introduces the bond factor

$\beta \quad = {}^{137} / {}_{136}$.

He deduces the fine structure constant is exactly $\alpha^{-1} \quad = 137$ *see note* ①

He also hypothesises the number of particles in the universe

$N = {}^{3}/_{2} \times 136 \times 2^{256}$ *see note* ②

To summarise the N in EU is the number of protons only N_p but the N in FT is the total number of particles so N_T. Eddington assumes $N_T \cong 2N_P$

Hereon N may be assumed to be N_T

The first calculation is the force constant $\mathcal{N} = 2\sqrt{(5N)} / 3\pi\beta^2$

$= 2.27140E+39$

The uncertainty constant he calculates as $\sigma = 3 / (136^2.137.16\pi\sqrt{5}R)$

$= 9.60400E{-}14\ cm^{-1}$

which is identical to

$3\sqrt{({}^{4N}/_{5})} / 32.136^2.137.R.\pi.\sqrt{N}$

The uncertainty constant depends only on the measured value of the Ryberg constant as N_T is presumed an exact value.

The nuclear range constant

$k = 2\sigma = 1.92080E{-}13\ cm^{-1}$

and this equals $R_0 / \sqrt{N}$

where R_0 is the Einstein radius.

So $R_0 = 2\sigma\sqrt{N} = 9.33544E+26$

The pseudo steady state volume of the Einstein universe, a 4 dimensional hypersphere is determined by

$V_0 = 2\pi^2 R_0^3 = 1.60596E+82\ cm^3$

Relative speed of recession is given by

$v_0 = c / R_0\sqrt{3}\ sec^{-1}$

and the age of the universe by

$T_0 = 1 / v_0 = 5.39061E+16\ sec$

≈ 1.7 billion years

which is far too low. There is not much scope to increase to a more realistic value because it derives purely from the measured value R and a predetermined value N_T.

Eddington himself is aware of this problem as he talks in EU about the "time-grabbing" evolutionist. But let's press on

The radius of curvature is

$R_s = c / v_0 = 1.61695E+27\ cm$

The Einstein radius can also be given as

$R_s = \sqrt{3}\ R_0$

and he thus calculates the Hubble constant as $H_0 = v_0 \times k_3 / 10^5$ where $k_3 = 3.08568E+24$ cm in a megaparsec

Hence Hubble's constant becomes

$H_0 = 572$ km/sec/megaparsec

Now he comes to the cosmological constant

$\lambda \quad = 1 / R_0^2 = 1.14744E{-}54\ cm^{-2}$

He also recalculates the uncertainty constant $\sigma = R_0 / 2\sqrt{N}$

$= 9.60396E{-}14\ cm$

Finally the constant of gravitation is given by

$G \quad = F'^2 c^2 . 136.137. \sqrt{(^9/_{20})}.\pi\beta^{1/4}.10\sqrt{N}$

$= 6.66650E{-}8\ cm^3/gm\ sec^2$

which compares only marginally well with the current measured value

$G \quad = 6.77206E{-}8\ cm^3/gm\ sec^2$

In the final chapter, reconstructed after his death but based on a lecture already given he finally gives an argument for $N_T = {}^3/_2 \times 136 \times 2^{256}$ but I regret there seems to be the "rabbit out of the hat" feeling in the last two lines.

Eddington must certainly have felt a degree of frustration that despite his best efforts T_0 is obstinately too low and G in poor agreement with the measured value.

Conclusions

Because Eddington is determined that the roots of his "equation of everything" give the mass values of proton and electron, he engages in some impressive algebraic manipulation which all had to be calculated by hand.

However he never seems to realise or acknowledge that the original starting equation already gives the appropriate ratio.

As later experimental investigations shift the fine structure constant from 136 to 37 he has to revisit and adjust with bond factors raised to various powers but never really improves on the result he had at the outset.

He does however cling to the cosmological constant throughout even though Einstein has labelled his greatest blunder which has now resumed prominence in the Einstein field equations.

Appendix A

Quadratic Equation roots m_p & m_e

Eddington wants to use 10 (from the 10 Einstein field equations) and 136 degrees of freedom. So I start with

$$10m^2 – 136\ m + 1 = 0$$

Dividing through by 10 will give an equivalent equation with the same roots. So $m^2 – {}^{136}/_{10}\ m + {}^{1}/_{10} = 0$

The sum of roots is $\alpha + \beta = {}^{-b} / {}_{a}$

and product of roots $\alpha\beta \quad = {}^{c} / {}_{a}$

so $m_p + m_e = {}^{136}/_{10}$

which is obviously too big so Eddington introduces the comparison particle m_0

So now I have $m_p + m_e = {}^{136}/_{10}\, m_o$

So as each value for m_p and m_e increases by the factor m_o^2 the product of the roots m_pm_e given by ${}^{c}/_{a}$ must increase by m_0^2

In 1931 Eddington settled on

$$10m^2 –136m_om + m_o^2$$

and I have three key relationships

$m_h \quad = m_p + m_e$

$m_h \quad = 136m_0 / 10$ (sum of roots)

$m_pm_e = m_o^2 / 10$ (product of roots)

Eddington then introduces the intracule mass

$\mu \quad = m_pm_e / (m_p + m_e)$

which is given by ${}^{-c} / {}_{b}$ or

$\mu \quad = m_0 / 136$

So $m_h / \mu \quad = 136^2 / 10 = 1849.6$

m_p and m_e are termed standard masses and are slightly different from current masses.

Appendix B

Fine structure constant

In SI units I start with

$\alpha = (1/4\pi\varepsilon_0)\ e^2 / hc$

where Coulomb's constant k_e is $(1/4\pi\varepsilon_0)$

However in cgs_{esu} $k_e = 1$ and so

$\alpha = e^2 / hc$ in cgs_{esu}

which is the expression used by Eddington. Wherever he uses 137 he means α^{-1}

Now as $c = 1 / \surd(\varepsilon_0\mu_0)$

I have $\varepsilon_0 = 1 / c^2\mu_0$

from Maxwell's electromagnetic theory

so $\alpha = (\mu_0 / 4\pi)\ e^2c / \hbar$

Now as Coulomb's constant is also given by $k_e = \mu_0c^2 / 4\pi$

I have $\alpha = k_e\ e^2 / \hbar c$

Finally the von Klitzing constant is given by $R_K = h / e^2$

Remembering $\hbar = h / 2\pi$

I deduce $\alpha = c\mu_0 / 2R_K$

The modern value for α^{-1} is 137.035999 …

Eddington was first convinced the value was 136. When measurements moved more to 137 he convinced himself there was a packing constant $^{137}/_{136}$ involved. Sadly this earned him the nickname Arthur Adding one and the broader criticism that he was just a numerologist

Appendix C

Brief Biography Arthur Eddington

(with acknowledgments to Wikipedia)

Sir Arthur Stanley Eddington OM FRS (28 December 1882 – 22 November 1944) was an English astronomer, physicist, and mathematician. He was also a philosopher and populariser of science. The Eddington limit, the natural limit to the luminosity of stars, or the radiation generated by accretion onto a compact object, is named in his honour.

Around 1920, he anticipated the discovery and mechanism of nuclear fusion processes in stars, in his paper "The Internal Constitution of the Stars". At that time, the source of stellar energy was a complete mystery; Eddington was the first to correctly speculate that the source was fusion of hydrogen into helium.

Eddington wrote a number of articles that announced and explained Einstein's theory of general relativity to the English-speaking world. He also conducted an expedition to observe the solar eclipse of 29 May 1919 that provided one of the earliest confirmations of general relativity and he became known for his popular expositions and interpretations of the theory.

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Appendix C Mathematical Theory of The Expanding Universe

Speed of recession of distance objects H_0 is given as 528 km / sec / megaparsec

Speed of light c is given as 2.99776E+10 cm / sec

Light year $c' = c \times$ secs in a year = 9.46021E+17 cm

Astronomical unit k_1 by definition is exactly 1.49598E+13 cm

A megaparsec k_2 by definition is exactly 2.06265E+11 au's

cm in a megaparsec $k_3 = k_1 k_2$ = 3.08568E+24 cm

Light-years in a megaparsec $k_4 = k_3 / c'$ = 3.26174E+06 light-years

Relative speed of recession $v_0 = H_0 \times 10^5 / k_3 = 1.71243E{-}17\ \text{sec}^{-1}$

Radius of curvature of empty space $R_s = c / v_0$ = 1.75059E+27 cm

Initial radius of the Einstein universe $R_0 = R_s / \sqrt{3}$ = 1.01070E+27 cm

Initial radius of the Einstein universe $R_0' = R_0 / k^3$ = 328 megaparsecs

In FT R_0 is derived from the uncertainty constant which itself is calculated from Ryberg's constant

Speed recession of distant objects $H_0' = H_0 \times 10^5 / k_4\, c' = 1.71243E{-}17\ \text{sec}^{-1}$

Relative speed of recession = $v_0 = c / R_e\sqrt{3} = 1.71243E{-}17\ \text{sec}^{-1}$

Initial radius of the Einstein universe $R_0'' = R_0' \times k^4 / 10^6$ = 1068 million light years

Initial radius of the Einstein universe $R_0 = R_0'' \times c' \times 10^6$ = 1.01070E+27 cm

Radius of curvature of empty space $R_s = R_0 \sqrt{3}$ = 1.75059E+27 cm

Constant of gravitation G is given as 6.66000E–08 $\text{cm}^3 / \text{gram-sec}^2$

From $GM / c^2 = \frac{1}{2} \pi R_0$ we can calculate $M_e = \frac{1}{2}\pi R_e c^2 / G$ = 2.14E+55 grams

Mass sun M_s is given as 1.98900E+33 grams

Einstein Universe in sun masses is $M_e' = M_e / M_s$ = 1.08E+22 suns

Now $M_0 \approx N m_p$ so $m_p = M_0 / N$ = 1.66E–24 grams

Then $N = M_e / m_p$ 1.29E+79 (number of protons in the universe)

From $R_s / \sqrt{N} = e^2 / m_e c^2$ we have $N = (R_s \times m_e c^2 / e^2)^2$ = 9.37121E+78 ie right order

From $N / R_s = \frac{1}{2}\pi c^2 / \sqrt{3} G m_p$ then $N = R_s\, \frac{1}{2}\pi c^2 / \sqrt{3}\, G m_p$ 1.29049E+79 which is exact

However we could calculate both R and N direct

From $R_s / \sqrt{N} = e^2 / m_e c^2\, R_s$ (say φ) = 2.85928E–13 cm

From $N / R_s = ½\pi c^2 / \sqrt{3}Gm_p$ then $N / R_s = ½\pi c^2 / \sqrt{3}\ Gm_p\ (\kappa)$ = 7.37175E+51 cm^{-1}

and a bit of algebraic manipulation gives $R_s = \varphi^2\kappa$ = 6.02676E+26 cm right order

and $N = \varphi^2\kappa^2$ = 4.44278E+78 cm right order

Volume spherical space radius R given by $V_e = 2\pi^2 R_s^{\ 3}$ = 2.03799E+82 cm^3

Initial mean density $\rho_e = M_e / V_e$ = 1.05E–27 grams / cm^3

That is one hydrogen atom per V_e / N = 1579 cm^3

Cosmological constant $\lambda = 1 / R_0$ = 9.8–55 cm^{-2}

Elementary charge e is given as 4.80480-10 franklins

Eddington wants e^2 / mc^2 to have units of length so he uses esu units of charge

Mass proton $M_0 \approx Nm_p$ = 1.66000E–24 grams

Mass electron $m_e \approx m_p$ / 1847.6 = 8.98463E–28 grams

Planck's constant h is given as 6.62610E–27 erg-sec (units of action)

From General Relativity $N / R_s = ½\pi\ c^2 / \sqrt{3}\ Gm_p$ = 7.37175E+51 cm^{-1}

and directly from values already calculated N / R_s is = 7.37175E+51 cm^{-1} agreeing

The central equation $R_s / \sqrt{N} = e^2/m_e c^2$ gives $R_s / \sqrt{N}$ = 4.87312E–13 cm

and $e^2 / m_e\ c^2$ = 2.85928E–13 cm same order

but if I insert the factor $\sqrt{3}$ I get $\sqrt{3}\ e^2 / m_e\ c^2$ = 4.95242–13 so within1.6%

The fine structure constant $\alpha^{-1} = hc / 2\pi e^2$ and is fixed at 137 exactly

And now I come to the six central equations

(A) $\alpha^{-1}\ m_e = \sqrt{N} / R$ gives $m_e = \alpha \sqrt{N} / R$ 1.49786E+10 natural units

(B) so now I have 136 $m_e = \sqrt{N} / R$ giving both sides equal to 2.05207E+12

(C) so now I get $136m_e - \sqrt{N} / R = 0$

Now I must put m_p into natural units by adjusting the natural units of m_e' by the mass ratio so $m_p' = m_e' \times (m_p / m_e)$ = 2.76745E+13

(D) the ratio sum roots / product roots 136 $m_p'\ m_e' / (m_p + m_e)$ = 2.03599E+12and $\sqrt{N} / R$ is 2.05207E+12 in close agreement

(E) Eddington's first attempt is quadratic $10\ m^2 - 136\ m + 1$

so a = 10, b = –136 and c = 1.00

So the root $\alpha = \{-b + \sqrt{(b^2 - 4ac)}\} / 2a$ and root $\beta = \{-b - \sqrt{(b^2 - 4ac}\} / 2a$

and the ratio $\alpha / \beta = 1847.6$ which is what Eddington is really after.

(F) Then in natural units I have $10\ m^2 - 136^{\sqrt{N}} /_{R}\ m + N/R^2$ where

$a = 10$, $b = -2.79E+14$ and $c = 4.211E+24$

And by the same method I obtain roots $m_p' = 2.78931E+13$ and $m_e' = 1.50969E+10$

And the ratio is 1847.6 again and $\delta_1 = 135.9246\ m_e = \sqrt{N}/R_s$ and

$\delta_2 = 0.073569\ m_p = \sqrt{N}/R_s$

These last three values are those given on p89 of the Expanding Universe and took me 50 years to achieve. The problem is δ_1 and δ_2 are only dependent on 10 and 136 and independent of N and R.

δ_1 will always be within 0.05% of α^{-1} with α^{-1} in the range 120 to 140

Appendix D Mathematical Theory of Fundamental Theory

If no units are specified the measure is dimensionless

Speed of light c is given as 2.99776E+10 cm / sec

Light-year is calculated as 9.46021E+17 cm

Rydberg constant ($^{Rc}/_{c}$) R is measured as 1.09678E+05 cm^{-1}

Rydberg constant ($^{Rc}/_{c}$) calculated as $R = 2\pi^2 m_e e^4/ch^3$ 1.09737E+05 cm^{-1} *so same order*

Here Eddington takes the mass of the electron yet to be derived from the final quadratic equation (29.6)

Faraday's Constant F' is measured as 9.57356E+03 abcoloumbs *(32:2/10) R13*

From F' I derive e' then e

Bond factor $\beta = {}^{137} / {}_{136}$ is calculated as 1.00735E+00 *(20:2) (32:2/6)*

Faraday's Constant $F = \beta^{1/24}F'$ is calculated as 9.57648E+03 abcoulombs *(32:3) R12*

Fine structure constant $\alpha^{-1} = hc / 2\pi e^2$ is defined as 137 exactly *(32:2/1) R15*

Having fixed e above from the measured value of F' above, this relationship now fixes h

Defining $\hbar = {}^{h}/{}_{2\pi}$ *gives* $\alpha^{-1} = \hbar c / e^2$ so 137 *from hereon represents* $\hbar c / e^2$

Particles in Universe $N = {}^{3}/_{2} \times 136 \times 2^{256}$ calculated 2.36216E+79 *(51:3) R17*

Force Constant $^{esu} / _{gravitational}$ $\mathcal{N}_{pe} = 2\sqrt{(5N)} / 3\pi\beta^2 = 2.27266E+39$ *(51:7) R23*

Force Constant $^{esu} / _{gravitational.}$ recalc $\mathcal{N}_{pe} = e^2 / Gm_p m_e = 2.27266E+39$ *(51:7) R23*

uncertainty constant $\sigma = 3 / (136^2.137.16\pi\sqrt{5}R) = 9.60396E–14$ cm *(4:1)*

or I express as $\sigma = 3\sqrt{(^{4N}/_{5})} / 32.136^2.137\ R\ \pi\ \sqrt{N} = 9.60396E–14$ cm *(agrees) (3:8)*

or alternatively as $\sigma = (R_0 / 2\sqrt{N})$ 9.60396E–14 cm *(agrees) (50:5) R27*

Nuclear range constant $k = 2\sigma = (R_0 / \sqrt{N}) = 1.92079E–13$cm *(5:2) (50:3) R24*

Einstein radius $R_o = 2\sqrt{N}\sigma$ is calculated as 9.33544E+26 cm *(3:8) R18*

Ratio R_0 / N is calculated as 3.95208E–53 cm

Ratio $k = R_0 / \sqrt{N}$ is calculated as 1.92079E–13 cm *(5:42)*

Volume Einstein Universe $V_0 = 2\pi^2 R_0^3$ calculated as 1.60596E+82 cm^3 *(24:0)*

Eddington's conversion figure f is calculated as 3.08732E+24 cm in a megaparsec

Einstein radius in megaparsecs $R_o = 2\sqrt{N}\sigma \div f$ = 3.02380E+02 megaparsecs *R19*

Relative speed of recession $v_0 = c / R_0\sqrt{3}$ = 1.85397E–17 sec^{-1} *(5:43) R22*

Age of universe $T_0 = 1/v_0$ is calculated as 5.39384E+16 sec

Age of universe T_0 seconds converted to billion years 1.70921E+00 billion years

In Expanding Universe Eddington discusses the "time grabbing evolutionist". This figure is far too low.

Radius of curvature $R_s = c / v_0$ is calculated as 1.61695E+27 cm

Hubble's constant $H_0 = f v_0 / 10^5$ is calculated as 5.72378E+02 km/sec per megaparsec

Ratio $m_p / m_e = \eta_1 = 136^2/10$ is calculated as 1.84960E+03 *(18:6)*

Einstein radius $R_o = R_s / \sqrt{3}$ is calculated as 9.33544E+26 cm

Cosmological constant $\lambda = 1 / R_0^2$ is calculated as 1.14744E–54 cm^{-2} *(39:70)*

Constant $G = F'^2c^2 136.137.\sqrt{(^9/_{20})}\pi\beta^{1/4}/10\sqrt{N}$ = 6.66648E–08 $cm^3/gram.sec^2$

(51:4)(51:6) R16

Mass Einstein Universe $M_0 = \pi R_o c^2 / 2G$ = 1.97675E+55 grams (5:3) *R20*

Density of Einstein universe $\rho_0 = M_0 / V_0$ 1.23089E–27 $grams/cm^3$ *R21*

Mass hydrogen atom $m_h = R_0 / N \times \pi c^2/G$ = 1.67368E–24 grams *(5:41) R1*

Mass hydrogen atom $m_h = m_p + m_e = 2M_0 / N$ = 1.67368E–24 grams *(agrees)* *R1*

I can now recalculate the two key ratios and I discover complete agreement with values above.

Ratio $k = R_0 / \sqrt{N} = (^{136}/_{10})\sqrt{(^9/_{20})}\beta^{1/6}h/2\pi c m_h$ = 1.92079E–13 cm *(agrees)* (51:2)

Ratio $R_0 / N = Gm_h / \pi c^2$ is calculated as 3.95208E–53 cm *(agrees)* *(51:1)*

In EU Eddington starts with these from other theoretical considerations and recovers R_0 and N

comparison particle $m_o = 10m_h / 136$ = 1.23065E–25 grams *(32:2/2) R2*

Mass hydrogen atom $m_h = {}^{136}/_{10}m_o$ = 1.67368E–24 grams *(agrees)*

molarly controlled charge $e' = F'm_h c$ = 4.80333E–10 franklins *(32;2/10) R7*

Elementary charge $e = \beta^{1/24}e'$ is calculated as 4.80480E–10 franklins *(32:2/4) R6*

Plank's constant $h = 2\pi 137e^2 / c$ is calculated as 6.62909E–27 erg-sec *(29:91) R8* molarly controlled h' = $\beta^{-1/12}$ h is calculated as 6.62504E–27 erg-sec (*agrees*) *(32:2/5)R9*

ratio h / e = $2\pi 137e / c$ is calculated as 1.37968E–17 volt-sec *R10*

Plank's constant $\hbar = 137e^2/c$ is calculated as 1.05505E–27 erg-sec *(29:91)*

Theoretical Plank's constant $\gamma = \hbar\sqrt{\eta_1}$ = 4.53746E–26 erg-sec *(40:4)*

Mass hydrogen atom $m_h = {}^{136}/_{10}\ {}^{3}/_{4}\ \beta^{1/6}\hbar\sqrt{({}^{4}/_{5}N)}/ 2cR_0$ 1.67368E–24 grams *(40:8)*

comparison particle recalc $m_o = \frac{3}{4}\beta^{1/6}\hbar\sqrt{({}^{4N}/_{5})} \div cR_0$ = 1.23065E–25 grams *(40:8)*

Ratio $R_0 / \sqrt{N}$ recalc = ${}^{136}/_{10}\ \sqrt{{}^{9}/_{20}}\ \ \beta^{1/6}\hbar / cm_h$ = 1.92079E–13 cm cm *(51:2)*

$\varpi = \hbar / 2\sigma$ is calculated as 5.49280E–15 erg-sec / cm *(38:72)*

Alternatively use $m_h = 136^2.2.137^2.R.h / (10\beta^{5/6}c)$ 1.67368E–24 grams *(agrees)*

Plank's constant $\hbar = 137F^2{m_h}^2c$ is calculated as 1.05505E–27 erg-sec *(agrees)* *(29:91)*

Faraday's Constant F ' = e' / m_hc is calculated as 9.57356E+03 abcoulombs *(agrees)*

Intracule mass $\mu' = m_o/_{136}$ 9.04887E–28 grams *(18:5)*

Intracule mass $\mu = m_pm_e / (m_p + m_e) = \beta^{5/6}m_o / 136$ 9.10429E–28 grams *(32:2/3) R3*

We can also recalculate the Rydberg constant substituting 137 for $\hbar$c / e^2 *and rearranging*

Rydberg constant $({}^{Rc}/_{c})$ $R = \frac{1}{2}\,({}^{1}/_{137})^2\,\mu\, c / h$ = 1.09678E+05 cm^{-1}*(agrees)*

(29:1) (32:2/9)

Ratio m_p / m_e $\eta_1 = m_h / \mu$ 1.84960E+03 *(18:6)*

Product current masses $m_pm_e = \beta^{5/6}{m_0}^2/10$ = 1.52377–51 gram^2 *(29:6)*

Product current masses $m_pm_e = 3\pi\beta^2e^2 / 2\kappa\sqrt{5N}$ = 1.52377E–51 gram^2 *(agrees)*

Sum current masses $m_h = m_p+m_e = {}^{136}/_{10}m_o$ = 1.67368–24 grams *(18:5) (29:7)*

and as a check I can now recalculate the intracule mass μ *directly.*

$\mu = m_pm_e / m_p+m_e$ = 9.10429E–28 grams (*agrees*) *(32:2/8)*

Appendix E Eddington's Quadratic Equation

A quadratic in the form $m^2 - (m_p + m_e)m + m_p m_e$ has roots m_p and m_e.

For quadratic $\mathbf{10m^2 - 136m_0 m + b^{5/6} m_0^2}$ *(29.6)*

comparing to the standard form $am^2 + bm + c$ then a = 10, b= $^-36m_0$ and c = $b^{5/6}m_0^2$

The roots are α (m_p) = 1.67277E–24 and β (m_e) = 9.10924E–28

and $m_p + m_e$ = 1.67368E–24 which agrees with the calculated results in Appendix D.

Finally m_p / m_e = 1836.341984. m_p and m_e are the standard masses to be distinguished from the current masses m_p' and m_e' .

Footnotes

①

P227 "The Anthropic Cosmological Principle" by Barrow and Tipler states

136 is the number of terms in a 16-dimensional tensor

$\frac{1}{2}(16^2 - 16) + 16 = 136$

to which Eddington later adds 1 by invoking a dubious concept of "packing density"

②

During a course of lectures he delivered in 1938 as Tarner Lecturer at Trinity College, Cambridge, Eddington said

"I believe there are 15 747 136 275 002 577 605 653 961 181 555 468 044 717 914 527 116 709 366 231 425 076 185 631 031 296 protons in the universe and the same number of electrons"

which is equivalent to $N_p = 136 \times 2^{256}$

Volume 40 Issue 1 of the Proceedings of the Cambridge Philosophical Society, written by Eddington is titled "The Evaluation of the Cosmical Number" and was subsequently appended to the posthumous publication "Fundamental Theory".

Here he deduces that $N_T = {}^3/_2 \cdot 136 \cdot 2^{256}$.

The philosophy eludes me but even at the end it seems he cannot quite reconcile whether N is N_T *or* N_P *and finally settles for a compromise mean.*