Zak Hunter Dunn's First Little Book on

Fibonacci Numbers

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Introduction

Fibonacci numbers are

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, …

Each number is the sum of the two previous numbers.

True or False?

Every 3^{rd} term is divisible by 2.

Every 4^{th} term is divisible by 3.

Every 5^{th} term is divisible by 5.

Every 6^{th} term is divisible by 8.

Every 7^{th} term is divisible by 13.

Every 8^{th} term is divisible by 21.

What about the 9^{th} term?

Fibonacci Triangles

Is it possible to construct a triangle with each side a different Fibonacci number?

Try This One

Take any three consecutive Fibonacci numbers.

Multiply first and third together.

Square the second number.

What's the difference?

Is this always true?

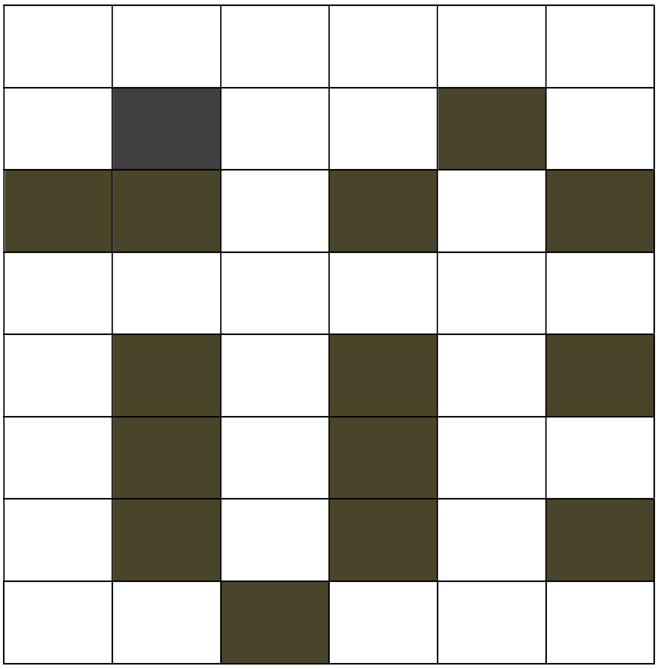
One Million

Is one million a Fibonacci number?

Odd?

How many of the first 999 Fibonacci numbers are odd?

The Fibonacci Cross Number



Every number (more than one digit) across or down is a Fibonacci number. Complete the grid.

Even Sums

Sum the even terms of the Fibonacci sequence.

1 + 3

1 + 3 + 8

1 + 3 + 8 + 21

etc.

Odd Sums

Sum the odd terms of the Fibonacci sequence.

1 + 2

1 + 2 + 5

1 + 2 + 5 + 13 etc.

In each case what patterns do you notice?

Digit Sum

Reduce each Fibonacci number to its digit sum.
Is there a pattern?

Pascal's Triangle

The Fibonacci numbers are "hidden" (well not that hidden) in Pascal's triangle. Find them!

The Golden Ratio

Take each term of the Fibonacci sequence and divide it by the term preceding it. (A calculator might help).
What do you notice?

Super Fibonacci

Turn the Fibonacci sequence into a series

1 1 2 3 5 8 13 21 34 55 89 144

1 2 4 7 12 20 33 54 88

Is it possible to calculate the next term of the sequence directly from the terms of that sequence itself?

Lagrange's Discovery

Find the remainder of each term after dividing by 4. What was the pattern Lagrange discovered?

Sign of Four

Take any four consecutive Fibonacci numbers.

Multiply the two outside numbers together.

Multiply the inside numbers together.

What is the difference of the two results?

Does your rule always hold?

Square Sums

Square each term of the Fibonacci sequence and sum them and then factorise the answer.

What do you notice?

$1^2 = \_ = \_ \times \_$

$1^2 + 1^2 = \_ = \_ \times \_$

$1^2 + 1^2 + 2^2 = \_ = \_ \times \_$

$1^2 + 1^2 + 2^2 + 3^2 = \_ = \_ \times \_$

$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = \_ = \_ \times \_$

$1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = \_ = \_ \times \_$

Consecutive Square Sum

Square each term of the Fibonacci sequence.

Sum each consecutive pair of squares.

$1^2 + 1^2 =$

$1^2 + 2^2 =$

$2^2 + 3^2 =$

$3^2 + 5^2 =$

$5^2 + 8^2 =$

$8^2 + 13^2 =$

Is there a pattern in the answers?

Cubes

$2^3 + 1^3 - 1^3 =$

$3^3 + 2^3 - 1^3 =$

$5^3 + 3^3 - 2^3 =$

$8^3 + 5^3 - 3^3 =$

$13^3 + 8^3 - 5^3 =$

$21^3 + 13^3 - 8^3 =$

Search for a pattern in the answers.

Number Digits

The number of Fibonacci numbers with

only one digit is ___ .

only two digits is ___ .

only three digits is ___ .

only four digits is ___ .

only five digits is ___ .

only six digits is ___ .

only seven digits is ___ .

There must be a pattern.

On the Staircase

When you walk up a staircase you can either take one step at a time or two steps at a time.

How many different ways are there of climbing a

a one step staircase?

a two step staircase?

a three step staircase

a four step staircase?

a five step staircase?

Can you explain your answer?

Endless Decimal Sum

Add up this decimal sum.

0.0

0.01

0.001

0.0002

0.00003

0.000005

0.0000008

0.00000013

0.000000021

0.0000000034

0.00000000055

0.000000000089

0.0000000000144

0.00000000000233

etc.

Endless Decimal

Write the fraction $^1/_{89}$ as a decimal to 12 decimal places.

Paying Your Bills

Using only 1p and 2p coins how many different ways can you pay a

1p bill / 2p bill / 3p bill / 4p bill / 5p bill etc.

Appendix

The Fibonacci numbers are connected to the number ϕ (phi) defined as

$$\phi = \tfrac{1}{2}(1 + \sqrt{5}) = 1.618034\ldots$$

The importance of the number f in the theory of the Fibonacci numbers is illustrated by the expression.

$$F_n = {}^1/_{\sqrt{5}}\{\tfrac{1}{2}(1+\sqrt{5})\}^n - {}^1/_{\sqrt{5}}\{\tfrac{1}{2}(1-\sqrt{5})\}^n$$

For each value of a, this formula involving $\phi = \tfrac{1}{2}(1+\sqrt{5})$ and its conjugate $\tfrac{1}{2}(1-\sqrt{5})$ simplifies to give the integer value F_n.

For large n

$$F_n \approx {}^1/_{\sqrt{5}}\{\tfrac{1}{2}(1+\sqrt{5})\}^n$$

and so $F_n + {}^1/_{F_n} \approx \tfrac{1}{2}(1+\sqrt{5})\}^{n+1} + \{{}^2/_{1+\sqrt{5}}\}^n$

$= \tfrac{1}{2}(1+\sqrt{5})$ ie ϕ as before.

The number ϕ occurs as the positive root of the equation $x^2 - x - 1 = 0$

Thus $\phi^2 - \phi - 1 = 0$

So $\phi - 1 = {}^1/_{\phi}$

The ratio $\phi : 1$ is called the golden ratio and can be used to construct the golden rectangle having its edges in the ratio $\phi : 1$.

Answers

Warning : May contain errors and omissions

True or False?

As far as I can ascertain, the answer is "Yes" but I've yet to prove it.

The sequence stops at the 8th term divisible by 7.

Fibonacci Triangles

No because the two shorter sides must add up to more than the longer side. However there are a number of Fibonacci related triangles – look it up on the internet if you're interested.

Try This One

The difference is always 1. This fact leads to an interesting visual "lost square" illusion where squares are cut up and rearranged into rectangles.

One Million

One million is not a Fibonacci number?

We go from 832040 (30th) to 1 346 269 (31st).

Of these 21 are odd because the sequence is

odd odd even odd odd even etc.

The Fibonacci Cross Number

1	2	1	3	9	3
3	[redacted]	3	4	[redacted]	4
[redacted]	[redacted]	4	[redacted]	3	[redacted]
1	9	6	4	1	8
7	[redacted]	2	[redacted]	7	[redacted]
7	[redacted]	6	[redacted]	8	9
1	[redacted]	9	[redacted]	1	[redacted]
1	3	[redacted]	6	1	**0**

All Sum

The sum of n odd terms is F_{2n}

The sum of n even terms is $F_{2n+1} - 1$

Digit Sum

You get a sequence of 24 numbers which then repeats.

Pascal's Triangle

The numbers on the diagonals add up to give the Fibonacci sequence.

The Golden Ratio

The terms converge to the golden ratio.

Super Fibonacci

This series is 1 less than the Fibonacci series itself so just add the last two terms plus 1. That is

$\sum_{i=1}^{n} F_i = F_{n+2} - 1$

Lagrange's Discovery

The pattern repeats 1 1 2 3 1 0

Sign of Four

The result alternates plus 1 minus 1 and seems to hold indefinitely.

Square Sums

The results factorise into adjacent Fibonacci numbers.

Consecutive Square Sum

The answers are the odd Fibonacci numbers.

Cubes

The results are every 3rd Fibonacci number.

Number Digits

After the first four terms (6 5 5 4) I conjecture the pattern is

5 5 5 4 5 5 5 5 4 5 5 5 5 4

and then repeating.

Given that F_n is round $\phi^n/\sqrt{5}$ and using logs to determine the number of digits we have

No. digits F_n

= rounddown ($n\log\phi - \frac{1}{2}\log 5$) + 1

On the Staircase

The number of different ways are the Fibonacci numbers. But how to explain?

Endless Decimal Sum / Endless Decimal

Incredibly 1/89 gives you the endless decimal sum. This was only discovered in 1994 by Cody Birsner, a student at the University of Oklahoma. There is a neat proof on the internet using matrix series.

Paying Your Bills

The answers are presumably Fibonacci numbers

Historical Footnote

When the euro was first introduced in Britain the conversion rate was indeed ϕ.

A schoolboy decided to buy a CD in euros but one shop mis-entered the conversion rate as $\phi - 1$. So he bounced between two shops for about 3 purchases before the shop twigged what was happening.

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