

Further Trig Relationships

sin x

$$\begin{aligned}
 & \{ 1 + \sin x \} / \{ 1 - \sin x \} \\
 &= \{ 1 + \sin x \} \{ 1 + \sin x \} / \{ 1 - \sin x \} \{ 1 + \sin x \} \\
 &= \{ 1 + 2\sin x + \sin^2 x \} / \{ 1 - \sin^2 x \} \\
 &= \{ 1 + 2\sin x + \sin^2 x \} / \{ \cos^2 x \} \\
 &= \sec^2 x + 2 \sec x \tan x + \tan^2 x \\
 &= \{ \sec x + \tan x \}^2
 \end{aligned}$$

cos x

$$\begin{aligned}
 & \{ 1 + \cos x \} / \{ 1 - \cos x \} \\
 &= \{ 1 + \cos x \} \{ 1 + \cos x \} / \{ 1 - \cos x \} \{ 1 + \cos x \} \\
 &= \{ 1 + 2\cos x + \cos^2 x \} / \{ \sin^2 x \} \\
 &= \operatorname{cosec}^2 x + 2\cot x \operatorname{cosec} x + \cot^2 x \\
 &= \{ \operatorname{cosec} x + \cot x \}^2
 \end{aligned}$$

sinh x

$$\begin{aligned}
 & \{ 1 + i \sinh x \} / \{ 1 - i \sinh x \} \\
 &= \{ 1 + i \sinh x \} \{ 1 + i \sinh x \} / \{ 1 - i \sinh x \} \{ 1 + i \sinh x \} \\
 &= \{ 1 + 2 i \sinh x - \sinh^2 x \} / \{ 1 + \sinh^2 x \} \\
 &= \{ 1 + 2 i \sinh x - \sinh^2 x \} / \{ \cosh^2 x \} \\
 &= \operatorname{sech}^2 x + 2i \operatorname{sech} x \tanh x + \tanh^2 x \\
 &= \{ \operatorname{sech} x + i \tanh x \}^2
 \end{aligned}$$

cosh x

$$\begin{aligned}
 & \{ 1 + \cosh x \} / \{ 1 - \cosh x \} \\
 &= \{ 1 + \cosh x \} \{ 1 + \cosh x \} / \{ 1 - \cosh x \} \{ 1 + \cosh x \} \\
 &= \{ 1 + 2 \cosh x + \cosh^2 x \} / \{ 1 - \cosh^2 x \} \\
 &= -\operatorname{cosech}^2 x - 2 \coth x \operatorname{cosech} x \\
 &\quad \quad \quad - \coth^2 x \\
 &= - \{ \operatorname{cosech} x + \coth x \}^2
 \end{aligned}$$

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