Hexagon Flowers

To paraphrase Eric Laithwaite, he said during his famous Christmas Lecture that there is nothing duller than a teacher repeating the same old experiment to a bored class. Teachers need to be bold and have confidence that to know the outcome is not necessarily the prime requisite. So there was a unexpected surprise during a simple Mathematics investigation into patterns.

I was teaching my Intermediate group to deduce functions from a table connecting two variables. I wanted then to make up their own patterns on squared, hexagonal, or triangular paper. I convinced them that any regular pattern would reveal some underlying structure. As a backstop, I suggested "Hexagon Flowers" a favourite in Year 8 and good for anyone looking for a display.

Students draw successive rings of hexagons around a single starting hexagon and measure perimeter and area. I always assumed that perimeters would follow a linear pattern and area some quadratic.

The results for perimeters come out as follows

n	0	1	2	3	4	5
p		6	18	30	42	54
1st diff.			12	12	12	12

I teach my students to deduce the value at n = 0. Even though you can't draw the 0th hexagon, that value will give you the constant. In this case $p = 12n - 6$

Moving on to area, if we measure the total area, we get

n	0	1	2	3	4	5
a_t		1	7	19	37	61
1st diff.			6	12	18	24
2nd diff.				6	6	6

I teach several ways to deduce the function from the table. The easiest to understand is to divide the 2nd difference by 2 to find the coefficient of n^2. In this case we have 6 so the n squared term is $3n^2$.

We then deduct this term from the original values.

n	0	1	2	3	4	5
a_t		1	7	19	37	61
$3n^2$		3	12	27	48	75
Net		–2	–5	–8	–11	–14

As the values are decreasing by 3, this gives us the linear term. The value at n = 0 gives us the constant as before, so we now have $a_t = 3\,n^2 - 3\,n + 1$

or more neatly $a_t = 3\,n\,(n - 1) + 1$

But this day a student decides not to record the total area of each successive hexagon flower, but just the area of each successive ring, to produce

n	0	1	2	3	4	5
a_i		1	6	12	18	24
1st diff.		5	6	6	6	

Where's teacher's promise now that there will always be a regular pattern? This calls for a bit of hasty thinking.

Now the science department might demur but as a mathematician, let's reject the problem result and reconstruct our table, taking the first true ring as n = 1.

n	0	1	2	3	4	5
a_i	0	6	12	18	24	
1st diff.	6	6	6	6		

Now that looks more acceptable. It just remains to reconstruct the original total area table and discount that first solitary hexagon.

n	0	1	2	3	4
a_t	0	6	18	36	60
1st diff.	6	12	18	24	
2nd diff.			6	6	6

and by the same method we deduce

$$a_t = 3\,n\,(n + 1)$$

which has the merit of looking even tidier than our first effort.

The problem lies in that first hexagon drawn. In fact it isn't a hexagon ring at all – it's just a template for the first true ring. Mathematics spots the error and gives us the correct equation for the total area. A neat exercise that kept a teacher on his toes. *rg*