Implication Issue 1.1

My wife makes a rule "If it's likely to rain then I take my umbrella".

In propositional calculus we say **p** = "It's likely to rain" and **q** ="I take my umbrella"

The conditional statement may now be written $\mathbf{p} \Rightarrow \mathbf{q}$.

Actually I quite like my umbrella so I take it all the time but my wife says I look silly. She said I should only carry it when rain is likely. So she added a new rule.

"If it's not likely to rain then I don't take my umbrella" or $\neg \mathbf{p} \Rightarrow \neg \mathbf{q}$.

This is the **inverse** of the original statement but it does not mean the same thing. However putting the two rules together gives us the **biconditional** – "I can take my umbrella if and only if it's likely to rain" or $\mathbf{p} \Leftrightarrow \mathbf{q}$.

There is one other construct call the **converse** which would be

"If I carry my umbrella then it's likely to rain" or $\mathbf{q} \Rightarrow \mathbf{p}$

Here one must carefully note that there is no causal relationship here – it's not the taking of the umbrella that causes the rain; simply that a sharp eyed neighbour might observe my meticulous habits and use this rule as his personal weather forecast.

By examining the truth tables we discover the **inverse** and **converse** are identical but have a different meaning to the original implication. Finally by applying both the **inverse** and **converse** at the same time to an implication we obtain the **contrapositive** which does have an identical meaning to the original. So to summarise we have

Implication	"If it's likely to rain then I take my umbrella"	$\mathbf{p} \Rightarrow \mathbf{q}$
Inverse	"If it's not likely to rain then I don't take my umbrella"	$\neg \mathbf{p} \Rightarrow \neg \mathbf{q}$
Converse	"If I take my umbrella then it is likely to rain"	$\mathbf{q} \Rightarrow \mathbf{p}$
Biconditional	"I take my umbrella if and only if it's likely to rain"	$\mathbf{p} \Leftrightarrow \mathbf{q}$
Contrapositive	"If I don't take my umbrella then it's not likely to rain"	$\neg \mathbf{q} \Rightarrow \neg \mathbf{p}$

Lest you be in any doubt about all this, the truth tables soon establish that inverse and converse have identical meanings and the contrapositive has the same meaning as the original implication. See if your English teacher agrees.

p	**q**	**¬p**	**¬q**	$\mathbf{p} \Rightarrow \mathbf{q}$	$\mathbf{p} \Leftrightarrow \mathbf{q}$	$\neg \mathbf{p} \Rightarrow \neg \mathbf{q}$	$\mathbf{q} \Rightarrow \mathbf{p}$	$\neg \mathbf{q} \Rightarrow \neg \mathbf{p}$
T	T	F	F	T	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T	T

Note that the implication truth tables stubbornly always remain three **T**rues and a **F**alse because the logic is indifferent to the phraseology.

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