Detailed Examination of the Implicit Function   Issue 2

Holmes turned over the piece of paper and examined it with his magnifying glass.

"See here Watson where the pens strokes don't join – clearly this is a forgery"

"Brilliant, Holmes, but who's the culprit, the Duchess or the Gamekeeper?"

"I suspect the Duchess but the proof will be a small ink stain on her little finger – see the writing is smudged here".

(n.b. I made this up – this doesn't occur in any Holmes story)

If only it were so easy. Fortunately the Duchess confessed immediately avoiding the need for a trial where any judge would have thrown out the "evidence". But let's examine in more detail what's happening here.

A Truth Table for two statements A and B gives all the outcomes for truth or falsehood of each statement.

p	**q**	**v**
T	T	T
T	F	T
F	T	T
F	F	F

So the OR function is TRUE if either **p** or **q** is TRUE. and only FALSE if both **p** and **q** are FALSE

Now **p** $\Rightarrow$ **q** means **If p then q**. But **q** is not dependent solely on **p**. So the truth table is

p	**q**	**p** $\Rightarrow$ **q**
T	T	T
T	F	F
F	T	T
F	F	T

The Implicit function is only FALSE if it specifically contradicts the premise that **p** is TRUE when **p** is FALSE. In all other situations the Function is TRUE.

Now **p** (forged the letter) is the major premise and **q** (has ink on fingers) the minor premise. But in our rush for the truth, seeing ink on the fingers gets us putting "the cart before the horse". It's safer to try and reconstruct the Implicit function in terms of the OR function. The OR function gives us three TRUTHs for one FALSEHOOD just as the Implicit function does so it's just a question of judicious rearrangement. Leaving the outcome sequence as it is (T F T T) we just need to reverse the **p** sequence from T T F F to F F T T.

That is we set up **¬p** (not **p**). So now it's

¬p	**q**	**¬p v q**	So saying
F	T	T	"If they forged the letter then they have ink on their fingers"
F	F	F	is the same as saying
T	T	T	Either they didn't forge the letter or they have ink on their fingers OR BOTH.
T	F	T	It's the OR BOTH that determines ink on your fingers doesn't necessarily mean you're guilty.

Hofstadter calls this the Switcheroo Rule after Q. Q. Switcheroo the Albanian railroad engineer who did Logic on the siding. The correct term for this interchangeability is the much less prosaic Implication Rule.

To keep the same outcome sequence T F T T we can also switch around **p** and **q** around and negate them

¬q	**¬p**	**¬q** $\Rightarrow$**¬p**	So saying
F	F	T	"If they forged the letter then they have ink on their fingers"
T	F	F	is the same as saying
F	T	T	"If they don't have ink on their fingers then they didn't forge the letter."
T	T	T	but the latter is better at encapsulating all that the sentence really has to say.

For once the double negative (usually to be avoided) is much less likely to be misinterpreted. However it's less interesting in crime novels to deduce the innocent rather than nail the guilty.

The correct term for this interchangeability is the Transposition Rule.

And that's about it for the Implicit function.

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