Introduction to Logic Issue 1.1

Logic is the formal analysis of language. Its purpose is to eliminate any misunderstandings in the language itself and to be able to follow accurately and without error the precise meaning from one sentence to the next. The process started with Aristotle who reputedly did not like Mathematics so for two thousand years tended to be the poor relation.

At the start of the twentieth century there was a move, first propounded by Hilbert, to bring Mathematics under Logic - that is to devise a scheme where all statements in Mathematics could be determined to be true or false by following a prescribed method - in short to eliminate the thinking from mathematics.

The first major step was the creation of propositional calculus by Gottfried Leibniz. This scheme was not powerful enough to include arithmetic. The next step was the creation of the more powerful Predicate Calculus by Gottlob Frege, allowing sentence logic to include ordinary arithmetic.

At this stage two major problems arose. Frege was completing a monumental work in three volumes designed to put mathematics on a new logical footing. As the final book was going to print, Russell sent him a paradox - he had managed to formulate the English sentence "I am lying" into the mathematical language of sets that Frege was creating.

The next problem arose when Gödel started to develop the idea that just as English can talk about itself in the metalanguage, mathematics could be constructed at two levels - the language itself and metamathematics where mathematics talks about mathematics. From this stage, with some ingenuity he was able to construct a mathematical statement which also said in the metalanguage "This formula is unprovable" - that is a statement denying its own truth.

The position today is that we must accept mathematics as either inconsistent or incomplete and if we choose the latter as the lesser of two evils there must be statements that can be generated that can be neither proved nor disproved within the system. Whether these statements are actually true or not is more in the realm of philosophy than mathematics because consistent systems can be constructed by assuming either their truth or falsehood.

There is a deeper second theorem that states formal systems such as arithmetic can prove their own consistency only if they are actually inconsistent. But there are many pitfalls when attempting to understand the full implication of Gödel's second theorem, so best not to get too hung up about it *rg* ∞