Inverse Circular and Hyperbolic Functions

cos iy

Let cos iy = b

However cos iy = cosh y

cosh y = b

y = $\cosh^{-1}$ b

so cos (i $\cosh^{-1}$ b) = b

so i $\cosh^{-1}$ b = $\cos^{-1}$ b

and it immediately follows

i $\cosh^{-1}$ ($^1/_b$) = $\cos^{-1}$ ($^1/_b$)

so **i sech^{-1} b = $\sec^{-1}$ b**

sin iy

but sin iy = i sinh y

so i sinh y = b

y = $\sinh^{-1}$ ($^b/_I$)

sin (I $\sinh^{-1}$ ($^b/_i$) = b

i $\sinh^{-1}$ ($^b/_i$) = $\sin^{-1}$ b

Now if I multiply $^b/_i$ by $^i/_i$

I get ib. Because sinh h is an odd function it follows

– i $\sinh^{-1}$ ib = $\sin^{-1}$ b

and it immediately follows that

– i cosech^{-1} ($^1/_{ib}$) = cosec^{-1} ($^1/_b$)

and following through the same logic

I determine

i cosech^{-1} ib = cosec^{-1} b

tan iy

Let tan iy = b

but tan iy = i tanh y

so I tanh y = b

y = $\tanh^{-1}$ ($^b/_i$)

tan (i $\tanh^{-1}$ ($^b/_i$) = b

i $\tanh^{-1}$ ($^b/_i$) = $\tanh^{-1}$ b

and as tanh is also an odd function

i $\tanh^{-1}$ ib = $\tan^{-1}$ b

and it immediately follows that

– i $\coth^{-1}$ ($^1/_{ib}$) = $\cot^{-1}$ ($^1/_b$)

and by the same logic

i $\coth^{-1}$ ib = $\cot^{-1}$ b

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