Kaprekar Numbers

Introduction

D.R. Kaprekar was an enthusiastic Indian amateur mathematician who published the results of his investigations in number theory for many years around the nineteen forties and fifties. He is best remembered for his discovery of the Kaprekar process and Kaprekar's constants.

Having become fascinated by Kaprekar Constants it was often set as a challenge to the more enthusiastic maths students to find one before the internet really "took off" and made such challenges pointless.

The Kaprekar Process

Take any number. In the Kaprekar process we make two new numbers:

The highest number possible by writing the digits in descending order.

The lowest number possible by writing the digits in ascending order.

For example for a 3 digit number 381 gives

831 *(the highest possible)* and

138 *(the lowest possible)*

We then subtract the lower from the higher to give a new number.

so 831 – 138 = 693.

We then make two more numbers from 693 and repeat the process.

If any answer has fewer digits than you started with then add a "0" at the beginning and create two new numbers as before.

The K-Constant 3 Digit Numbers

For three digit numbers, all sequences end with the constant 495 because only

495 954 – 459 = 495

cycles back on itself.

The primary sequence *(expressed as the lower number of the two new numbers)* is

099 → 189 → 279 → 369 → 459Every three-digit number feeds into this primary sequence, but there is a beautiful pattern to it, depending on the first and last digit.

1, 10, 11 …	(54)	→ 099
9, 19, 29 …	(54)	→ 891
2, 12, 20 …	(96)	→ 198 → 792
8, 18, 28 …	(96)	→ 792
3, 13, 23 …	(126)	→ 297 → 693
7, 17, 27 …	(126)	→ 693
4, 14, 24 …	(144)	→ 396 → 594
6, 16, 26 …	(144)	→ 594
5, 15, 25 …	(150)	→ 495
111, 222, …	(9)	→ 495

The K-Constant 4 Digit Numbers

For four digit numbers, all sequences end with the constant 6174 because only

6174 -> 7641 – 1467 = 6174

cycles back on itself.

It is conjectured that all sequences converge onto this number in no more than six iterations.

The K-Constant 2 Digit Numbers

All two digit numbers feed into the cycle 09 > 18 > 36 > 27 > 45 > 09 > etc. so there is no specific constant.

The K-Constant 5 Digit Numbers

It is conjectured all numbers feed into one of three cycles

34479 > 24669 > 13779 > 23589

35559 > 45559

33579 > 34569 > 14679 > 22689

Postscript

The internet is a blessing and a curse.

Because I initially failed to find a 6 digit Kaprekar Number I assumed they did not exist above 5 digits. In fact it is easy to form subsequent even number of digits Kaprekar numbers from 6174.

Simply enter the same number of "3"s and "6"s into the brackets.

6()317()4

The K-Constant 6 digits

631764 → 766431 – 134667 = 631764

and hence a Kaprekar constant.

The K-Constant 8 digits

63317664 → 76664331 – 13346667

= 63317664

and hence a Kaprekar constant.

And the pattern continues for 10 digit, 12 digit, 14 digit numbers … etc

I do not know if there are other 8, 10, 12 … digit Kaprekar numbers.

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