

Lizzie's Method to Factorise Quadratics

Lizzie's method of factorising a quadratic is named after Lizzie Seagle, an American High School student who rediscovered a special case of Vieta's theorem. Here it is.

Suppose we are faced with $8x^2 + 10x + 3$. Assuming it will resolve into rational factors here's an algorithm to determine them.

$$ax^2 + bx + c \quad (nx + p)(mx + q)$$

though of course we don't yet know the factors

$$8x^2 + 10x + 3 \quad mnx^2 + (mp+nq)x + pq$$

Multiply c term by " a " term and set $a=1$

$$x^2 + bx + ac$$

$$x^2 + 10x + 24 \quad x^2 + (mp+nq)x + mnpq$$

find two factors of 24 that add to 10

$$24 = 2 \times 12 \text{ so (minus 2) and 12}$$

$$(x + 12)(x - 2) \quad (x + mp)(x + nq)$$

divide through by the " a " coefficient

$$(x + 12/8)(x - 2/8) \quad (x + mp/mn)(x + nq/mn)$$

because originally " a " was equal to mn

though we didn't know that at the time

$$(x + 3/2)(x - 1/4) \quad (x + p/n)(x + q/m)$$

Multiply 1st bracket by 2 and 2nd by 4

$$(2x + 3)(4x - 1) \quad (nx + p)(mx + q)$$

which is the correct result.

Here it is again without the explanation

$$12x^2 + x - 20 \quad \times \text{ last term by } 12$$

$$x^2 + x - 240 \quad \text{find factors } 240$$

$$(x + 16)(x - 15) \quad \text{divide by } 12$$

$$(x + 16/12)(x - 15/12) \quad \text{rationalise}$$

$$(x + 4/3)(x - 5/4) \quad \text{multiply out}$$

$$(3x + 4)(4x - 5) \quad \text{which is correct}$$

Summary

Still not got it?

This is what you do

- 1) Multiply the last coefficient by the first
- 2) Set the first coefficient to 1
- 3) Find two factors of the new last coefficient that add to the middle coefficient.
- 4) Set up the initial factorising expression
- 5) Now divide the second term in each bracket by the original first coefficient
- 6) Now rationalise that 2nd term
- 7) Now multiply through each bracket by the denominator of the 2nd term in each bracket.
- 8) And that's your answer.

Lizzie's method is harder to describe than actually undertake. It eliminates that stage of having to juggle with that first coefficient so eliminates it in a non-standard way but then sneaks it back in by another non-standard way. The two "cancel out" to restore a correct expression.

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