Think about the simplest motion detector light switch. You walk into the room and the light comes on automatically – but not in the middle of the day – only when it's dark. Inside the light switch is a simple logic gate with two inputs – say **p** which gives a signal when it detects light and **q** when it detects motion. So the logic gate is $\neg \mathbf{p} \wedge \mathbf{q}$. When there is no input from **p** and an input into **q** the light switches on. There is an extra bit of circuitry that stops the light from flashing on and off – once it comes on it stays on for a minimum period of time and often that can be changed as well. That's why you can be sitting quietly reading the paper and the light goes off and you have to wave your arms around to get it on again.

So how many different logic gates can there be for just two inputs? Well two inputs give us 4 possibilities – TT, TF, FT, FF, and for each of those 4 possibilities we can have T or F so in all there are 16 possibilities which we label as logic gates 0 to 15.

The first logic gate doesn't do much at all – whether we have an input on **p** or an input on **q** the logic gate remains stubbornly off (F). F F F F is called "denial" or "contradiction" and could be represented by $\neg \mathbf{p} \wedge \mathbf{p}$ or if you prefer $\neg \mathbf{q} \wedge \mathbf{q}$**.** It never happens. If you opened up the cover of logic gate 0 don't be surprised if there's nothing there at all.

At the other end we have logic gate 15 which isn't much better but at least has an effect – the light is always on no matter what the inputs **p** or **q**. For all 4 options TT TF FT FF the light stays on. We could express that as $\neg \mathbf{p} \vee \mathbf{p}$ or if you prefer $\neg \mathbf{q} \vee \mathbf{q}$**.** This is called a tautology – it's always true and the light is always on.

Now consider all the logic gates in between – numbers 1 to 14. Look at the attached handout. How many of them can you identify? Here are some hints.

Somewhere along the list you must find gate **p** and gate **¬p**, **q** and **¬q**. In fact using **p ¬q q ¬q** and connecting them with $\vee$ $\wedge$ $\Rightarrow$ or $\Leftrightarrow$ one might conclude there aren't enough gates – much more than 16 it would seem †. The answer is that de Morgan's rules and the Switcheroo rule mean that many combinations of **p ¬p q ¬q** joined by $\vee$ $\wedge$ $\Rightarrow$ **or** $\Leftrightarrow$ are equivalent.

Two gates, the NOR gate and the NAND gate are termed functionally complete – that is every one of the 16 possible gates can be built solely from these gates. This is because by linking the inputs of either of these gates they can be used as an inverter. It may be apocryphal but it's often quoted that the computer on Apollo 11 consisted of just 5600 NOR gates – which may seem a lot but your mobile phone probably contains several million.

† 32 in fact – can you see why?