Lorentz-Fitzgerald Contraction

Maths

Suppose we have a river velocity = v

and a swimmer with velocity = c

The swimmer swims to a point across the river and back but has to swim slightly into the current to maintain his/her bearing. So a vector diagram quickly demonstrates that the effective velocity

$c' = \sqrt{(c^2 - v^2)} = c\sqrt{(1 - v^2/c^2)}$

If the width is d the time to swim across and back is $t = 2d / c\sqrt{(1 - v^2/c^2)}$

Now assume there is a second swimmer who swims parallel to the bank to a post distance γd away such that that when swimming at the same speed the time taken is the same.

Upstream velocity = c – v

downstream velocity = c + v

So $\gamma d / (c - v) + \gamma d / (c + v)$

$= 2d / c\sqrt{(1 - v^2/c^2)}$

$\{\gamma d (c + v) + \gamma d (c - v)\} / \{c^2 - v^2\}$

$= 2d / \sqrt{(c^2 - v^2)}$

$2\gamma dc / \{c^2 - v^2\} = 2d / \sqrt{(c^2 - v^2)}$

$\gamma = \{c^2 - v^2\} / c\sqrt{(c^2 - v^2)}$

$= \sqrt{(1 - v^2/c^2)}$

This term is indeed known as the Lorenz-Fitzgerald contraction.

Discussion

These two gentlemen each tackled the conundrum of a nil result in the Michelson-Morley experiment – that is a split beam of light sent on two perpendicular paths and returned to the same point showed no fringe effects – even though one beam of light had presumably been transmitted against the ether wind. How could the two beams of light arrive back at the starting point still in phase?

Lorentz went back to Maxwell's equations for electromagnetism and determined what invariant quantity could be introduced to produce this effect.

Fitzgerald just said "By how much do we need to reduce the length of the arm facing into the ether wind to produce a null effect?" which is identical to the swimmer problem.

So Fitzgerald still assumed the ether existed and that it had a real physical effect on the length of any material moving against it.

So who's right? Debate.

∞ rg