Propositional Calculus meets Monk rather than Maigret

Propositional calculus (PC) uses letters to represent statements and then joins and manipulates those letters to form new expressions, which retain their common-sense interpretation.

But don't ever show this paper to a logician because he (or she) will still pick holes in it.

Statements

Statements are represented by (capital) letters.

p stands for (say) "Tom committed the crime"

q stands for "Tom went to the pub last Thursday"

¬q stands for "Tom **didn't** go to the pub last Thursday"

Be clear that statements can either be true or false. They start out as statements pure and simple.

Functions and Compound Sentences

Two statements can be combined together by a FUNCTION to form a compound sentence. The combination implies of itself that it's own "truth" (The Law of Identity). Whether the sentence in the final analysis is true or false depends on the truth or otherwise of the individual statements of which it is made following the rules of the connection functions.

^ (AND)

Tom is married AND has kids. **p ^ q**. Both have to be true for the compound sentence to be true.

If either or both are false the sentence is false.

If he isn't married or doesn't have kids then the whole expression is false.

v (inclusive OR)

Tom is either a thief OR a liar **p v q**. If either is true the compound sentence is true. Note that OR is inclusive.

Tom can be a thief AND a liar and the compound sentence is still true.

Initial Pitfalls

There are two key pitfalls in all this.

Tertium non datur

There is no third option. Either Dick went to the pub last Thursday or he didn't.

In PC **q v ¬q** is always true even if **q** is indeterminate.

This is termed two-valued logic.

If the TV detective is to be believed, all murder mysteries hinge on the detective discovering that the case is an example of three-valued logic

If Dick went to the pub he didn't commit the murder. Yes he did, he slipped out the toilet window, knocked the guy over the head and slipped back before anyone missed him.

Misinterpreting the Implication Rule

If Tom forged the letter he'll have ink on his fingers.

If **p** is "forging" and **q** is "ink on fingers" PC writes this as **p** ⇒ **q** (**if p then q**)

But **q** can be for other reasons (if it weren't we'd call this biconditional).

Tom's got ink on his fingers so he forged the letter.

No he didn't – he's got ink on his fingers because his pen leaks.

In PC certain expressions are interchangeable

p ⇒ **q** is interchangeable with **¬p v q**

If Tom forged the letter he'll have ink on his fingers

is the same as

Either Tom didn't forge the letter OR he has ink on his fingers (or most importantly both)

Which is exactly the case here.

Tom didn't forge the letter but he's also got ink on his fingers.

PC can show the false logic of assuming Tom forged the letter just because he's got ink on his fingers.

The full truth table is

p	**q**	**p⇒ q**	**p⇒ q^q**	**p⇒ q ^q ⇒ p**	
T	T	T	T	T	
T	F	F	F	T	
F	T	T	T	F	Here is the false logic!
F	F	T	F	T	

rg