The Monty Hall Dilemma

So how can you prevent parents sliding by your Mathematics Faculty table at a Year 8 parents' evening, either bemused by the vast array of increasingly complex calculators or simply not wanting to admit, "I never could do Maths at school myself"?

We needed a "hook" plus two keen Year 9 students, Izzie and Sophie. A few years back I came across the Month Hall Paradox, first talked about only in Maths meetings but now more familiar through books such as "The Curious Incident of the Dog in the Night-time" and the film "21".

Here it is. There are three doors (folders). Behind one is a car and behind the other two a goat (well an onion in our case – I couldn't find a picture of a goat). The corralled parent chooses a door. Now Sophie, who knows where the car is, opens one of the envelopes to reveal an onion. She always chooses an onion and this is clearly explained to the player as she does it. Now comes the challenge. Izzie offers the player the opportunity to switch their choice to the other remaining envelope.

Now we wanted to discover whether people in general are "stickers" or "switchers" and which is the better strategy in the long run? In fact we noted a few more aspects of human nature on the way.

Just the time taken is worthy of a separate investigation. Some chose instantly but the younger the participant the longer it seemed to take. For the students it was a just agonising decision involving anything from "eeny meeny miny mo" to an eventual appeal to the parent to help them decide. Some parents believing either it was an intelligence test or a magic trick would ask a range of detailed questions before deciding. We had to discount Mr. Cracknell, who immediately picked a folder, opened it before we could stop him and shrugged his shoulders saying, "lost again".

Of the thirty people who had a go, twenty-seven were "stickers". Wider trials suggest about 87% of the population

will stick so our lot appeared slightly stickier.

One student looked up at me after playing and asked almost pleadingly, "What one earth has this got to do with Maths?" So here's the official answer. Using a binomial test, the null hypothesis that people are equally "stickers" and "switchers" is soundly rejected. There is less than one chance in two million that we just happened to encounter 27 "stickers" that night on an assumed evenly split population.

How about success rates? Of the 27 stickers, 17 lost. That's about 63%., close to the expected value. Interestingly for the 10 "winner-stickers", it was always assumed to be a loss for a hypothetical switcher. This is quite a deep point – the "what-if" concept and often a dangerous assumption to make.

The basic argument used by the stickers is that faced now with two cards, the probability of winning must be 50% so I might as well stick to my first choice.

For the 3 switchers every one was a winner. We didn't investigate whether they were just indecisive or using the correct logic. There is a one in three chance of choosing the correct door. After a door is removed, the probability is unchanged if you stick. Therefore there is a two-thirds chance of winning if you switch, because the car has to be in one of the two remaining folders. Yes it's that simple but when the problem was first raised there were the most eminent mathematicians writing in on expensive college headed notepaper saying this was an incorrect result. Read all about it on Wikipedia.

The "chi-squared" test is used to decide if these results could have occurred by chance having accepted that most people are "stickers". That's our null hypothesis. Using one of the expensive calculators on display we can quickly read off the result – the null hypothesis may be rejected at 5% confidence level. There is actually about a 4% chance that the "stickers" were just unlucky and the switchers "lucky" on our limited trial

So all in all the results were close to the calculated and accepted values. Of course having hooked the parent there is then that twenty-second opportunity to explain

about the varied nature of Maths in Year 9 combining traditional grounding in core topics interspersed with challenging and thought provoking investigation – like this one.