Multigrades

Pick any three numbers say 1 and 5 and 7

Create equation 1 + 7 = 5 + 3

Add the difference of the first two numbers to each term *(in this case 6)*

(this is only to cancel out a number later on – adding any number will work)

So $7 + 13 = 11 + 9$

Remember we originally had

$1 + 7 = 5 + 3$

So by cross adding

$7 + 13 + 5 + 3 = 11 + 9 + 1 + 7$

and also (incredibly)

$7^2 + 13^2 + 5^2 + 3^2 = 11^2 + 9^2 + 1^2 + 7^2$

Cancelling out 7s gives

$13^2 + 5^2 + 3^2 = 11^2 + 9^2 + 1^2$

This is written as

$\mathbf{13 + 5 + 3 =^2 11 + 9 + 1}$

meaning both the linear sum and sum of squares are equal.

Start again with

$1 + 9 + 11 = 3 + 5 + 13$

Add 2

$3 + 11 + 13 = 5 + 7 + 15$

Cross add

$3 + 5 + 13 + 3 + 11 + 13$

$= 1 + 9 + 11 + 5 + 7 + 15$

$3 + 3 + 13 + 13 = 1 + 7 + 9 + 15$

and

$3^2 + 3^2 + 13^2 + 13^2 = 1^2 + 7^2 + 9^2 + 15^2$

Finally if

$1 + 6 + 8 =^2 2 + 4 + 9$

then

$12^2 + 64^2 + 89^2 =^2 21^2 + 46^2 + 98^2$

That such equations can be produced so easily defies intuition

Proof ?

Let $a + c = b + (a + c – b)$

add $(c – a)$ to each term both sides

$c + (2c – a) = (b + c – a) + (2b – c)$

cross add

$c + (2c – a) + b + (a + c – b) =$

$(b + c – a) + (2c – b) + a + c$

$(2c – a) + b + (a + c – b) =$

$(b + c – a) + (2c – b) + a$

ie $3c = 3c$ and as expected

But now square every term

LHS = $(2c – a)^2 + b^2 + (a + c – b)^2$

which if you multiply out reduces to

$2a^2 + 2b^2 + 5c^2 – 2ab – 2ac –2bc$

RHS = $(b + c – a)^2 + (2c – b)^2 + a^2$

which if you multiply out also reduces to

$2a^2 + 2b^2 + 5c^2 – 2ab – 2ac –2bc$

Hardly a proof but a demonstration of how it works – but not why.

Anyone have a better idea?

RG multigrades 04/00