Mr G's Little Booklet on

The Nature of Mathematics

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Preface

This was never intended for anyone to read. Sometimes I'd get a thought in my head and it just wouldn't go away and there were other things I needed to be getting on with – usually marking a pile of books. So the best way to get the topic out my head was to write it down.

This booklet is a collection of nine essays written between about 1998 and 2018. They are not in the order written, I've just rearranged them into hopefully a have logical narrative about my own personal view of nature, mathematics and God.

Isn't It Obvious?

Between 1910 and 1913 Bertrand Russell and A.N. Wilson published a mammoth three volume affair called "Principia Mathematica". It was an attempt to put all of mathematics then known on a sound logical base. One might suppose that such a work would contain theorems unrecognisable to most people. For certain, the content would certainly be incomprehensible – yet notwithstanding that, on page 83 of the second volume they finally get round to proving that 1 + 1 = 2. *(Yes, you haven't misread that.)*

Now there's something very odd going on here. We wouldn't be surprised that a piece of mathematics that looked impossibly complex then to be judged "simple" by a great mathematician. But here we have exactly the reverse - the simplest piece of mathematics that we could imagine being demonstrated to be " (almost) impossibly complex" by the greatest logician of the twentieth century. How could one possibly go about proving one plus one equals two, and taking more than one volume of tightly packed mathematics to get there – believe me, this is no two line affair?

The answer lies partly in our own brains. It would seem that some aspects are "hard-wired" into us and are so self-evident that no proof is necessary. We just accept it. The mathematics does not concern us.

Consider a slightly more complex situation. A parent sends a child to the larder to get a dozen eggs, but the child returns with only ten. When the mistake is pointed out, does the child return the ten eggs and then bring twelve? Well, possibly – it depends on the age of the child. Certainly "counting on" is an acquired skill – it isn't hard wired into us. Children acquire an intuitive feel for counting-on sometime in the early years of school.①

For higher level mathematics we often adopt strategies for giving us the solution. Perhaps a few people know that the first differential of x^3 is $3x^2$ by some rule of thumb – "we bring the 3 to the front and reduce the index by one" but fewer will know quite why this strategy works. ②

In teaching mathematics to students with additional needs we need to distinguish between genuine understanding and the use of a strategy that works "most times". A student might correctly identify the "mirror image" of a shape on four occasions then be completely wrong on the fifth. Why? Because the student was simply adopting an erroneous strategy – perhaps matching colours or counting squares for no better reason than it seemed to give the "right" answer.

Finally we should not be frustrated by the lack of understanding. You might think your home p.c. is wonderful, but take the cover off, swap over a couple of wires at random, and it would probably blow a fuse when next switched on.

The incredible aspect of the brain is that it continues to work at all even when it has suffered major damage – because it has designed into it a multiplicity of redundant circuits – continually finding alternative paths to complete the task – just like the Terminator at the end of "…Judgement Day".

But hiccups can occur. So some aspect of mathematics that seems so obvious to us, like identifying mirror reflections or "counting on" might remain hidden to a student with special needs. That's no different to our correctly hard-wired brains never appreciating that one plus one equals two is <u>not</u> a self-evident truth.

rg 18th March 1999

Goldbach's Conjecture

In 1742, an amateur mathematician sent a letter to Euler who was then at the Court of Frederick the great at Potsdam. He noticed that all even numbers (except 4) could be made from adding two (odd) primes together. He wanted to know if this worked for any number no matter how big and could it be proved. Euler took no interest in the problem at all, considering it trivial, but it has remained one of the great unsolved problems in mathematics.

No one made much progress until the Russian number theorist Schnirelmann proved that any number could be represented as the sum of not more than 300 000 primes! That seems such an incredibly useless and extremely likely result and one wonders where on earth the figure 300 000 comes from. Why so high? But no one could do any better until another Russian, Vinogradov, established that a sufficiently large even number could be represented as the sum of no more than four primes. So that result seemed a lot better than Vinogradov's but what did he mean by "sufficiently large". It meant that only numbers bigger than some value, say v, could be represented this way – but he could give no idea how big v might be.

So in recent years there have been two approaches –

- can 300 000 be reduced? and
- how big might v be?

Well the good news is 300 000 is now down to 6 – that is for any even number over 4 you won't need more than 6 primes.

As for that "sufficiently large" number v, a man called Brodzkin showed in 1937 that it wouldn't be bigger than $10^{7000000}$ (that's 1 followed by 7000000 zeros which is still ***big***) and two mathematicians, Chen and Wang have recently managed to get that down to 10^{7194}. With modern computing power it should soon be possible to check every even number up to that value and so immediately get the six prime number maximum down to four. Faber and Faber, the publishers have offered a $1 million reward for a solution to Goldbach's conjecture that you only need two primes.

The most famous unsolved problem in Maths is the Riemann Hypothesis, which is about proving a particular pattern does exist in all prime numbers. If that's solved, then the "four prime number" maximum is also immediately proved. (Don't ask why – it's too complicated). But that still leaves the field wide open to get it down to that elusive "two".

Don't waste too much time on it though. The smart money is on the fact that although in practice you never need more than two prime numbers, the best that could ever be proved is four.

There's another weird proof in mathematics that some things are true but not provable – and this conjecture could be one of them. But of course it will never be possible to prove that it's "true but not provable" because then you would have then proved it to be true – which can't be done!

rg 1st February 2000

The End of Mathematics

It's always exciting to read about the latest developments in Medicine or Physics but does anyone think the same happens in Mathematics. I suspect the majority think the whole subject was pretty well wrapped up by the ancient Greeks – most of the IGCSE textbooks would be comprehensible to an educated time-travelling Grecian

But take our two main Mathematical syllabuses here at Taunton School. We could be teaching "A" level Mathematics in D1 and in Standard Level IB next door in D2. Think of the simplest concept – the set **N** of counting numbers. Does that include "zero"? Well it's "No" if you're studying "A" level and "Yes" if you're studying IB. That's because the two are built on different axiomatic systems – the assumed unprovable truths.

For "A" level, it's Peano's axioms – the first one stating, "one is the lowest number" and the next three constructing the counting numbers. IB is built on set theory, with the second axiom (not even the first) being "There is a null set". That's {} or ϕ. So how many sets have we now got? Answer "one" and we're off first base with the counting numbers out of thin air.

Peano's axioms are self-evident, up to the fifth, when we get the axiom of induction. It's the axiom we use in those questions when we're given the answer and then asked to prove it. To me it always seemed to be a bit of a cheat. I mean if we have to be given the answer to start with – what's the point? Where did the answer come from? At university, when I was first shown how to solve differential equations, the lecturer said "Let's assume the solution is in the form Ae^{-bx}". So where did that come from? Later we were introduced to integrating factors but they also seemed to materialise out of nowhere. And so it goes on with summing series and many other areas.

At the turn of the twentieth century, set theory seemed to offer a whole new approach to mathematics – a new foundation based on more powerful if less intuitive axioms. However as Gottlob Frege was putting the final touches to a three-volume work, Bertrand Russell sent him a paradox of set theory – to do with sets being members of themselves. Frege wrote as an introduction to Volume 3 almost as it rolled off the press "A scientist can hardly meet with anything more undesirable than to have the foundation give way just as the work is finished."

Russell and Whitehead then embarked on a monumental work "Principia Mathematica" in an attempt to prop up the increasingly shaky structure of Set Theory. It was heavy going and took 362 pages before they managed to prove 1+1=2 (this is not a joke). As at today we take the modified axioms of Zermelo-Fraenkel as our foundation. There is even a specific axiom, the axiom of regularity that disallows sets being members of themselves, the cause of the original problem.

But then there is that tricky Axiom of Choice (AC). Take set A consisting of three objects {a,b,c}, set B consisting of {d,e,f) and set C consisting of {g,h,j). The axiom of choice says we can form a new different set D by choosing an element from each of the three sets. Say D = {a,d,g}. There is no way to derive this self-evident fact from the other axioms. The main problem is the axiom of choice doesn't tell us how to choose the elements so many mathematicians try and work without it. Two in particular, Tarski and Banach wanted to banish it altogether. Their best attempt was to assume it true and then they set out to prove the most ridiculous thing they could – which was that if you cut up an orange into six (some say seven) pieces, you can rearrange the pieces and re-assemble them into an orange twice as big – with no gaps! Unfortunately the message the rest of the (mathematical) world got was – "wow, isn't maths amazing" and we carry on using the axiom of choice whenever necessary.

Having said all that, no one in practical everyday mathematics ever pays any of this the slightest notice though some issues are touched upon in Higher Level IB (eg proving two sets are equal)

Mathematicians are supposed to fall into different camps. First we have the logicists led by Frege and Russell who ultimately decamped. The latter eventually wrote in "Portraits from Memory" "Having constructed an elephant upon which the mathematical world would rest, I found the elephant tottering and proceeded to construct a tortoise to keep the elephant from falling. But the tortoise was no more secure than the elephant."

Then there are the constructivists led by Brouwer. He starts with the natural numbers as "fundamental intuition" and everything from thereon has to be constructed. The problem is constructivist mathematicians are prevented from using some of the everyday tools as a sort of "matter of principle". For example there is the law of trichotomy – "every real number is either positive negative or zero". No one would argue with that and it can be proved if the numbers are constructed set-theoretically. Not so for the constructivists who reject it. This is because the proof depends upon "proof by contradiction" which itself rests upon the law of the excluded middle. I'd have to say I've never met a constructivist – I think they are as rare as solipsists (though I am one of the latter).

Then among others there are the formalists, started by David Hilbert at the turn of the century. He put out a challenge to base mathematics on a complete and consistent set of axioms from which all truths could be automatically derived by following the rules. It would be like turning the handle of the machine in Swift's island of Laputa – whereby all works could ultimately be written by assembling the random lists of words produced.

Gödel put paid to that in 1931 with an amazing paper now termed the incompleteness theorem. The bottom line developed further by the British mathematician Alan Turing was that mathematics could <u>never</u> be both consistent and complete.

Imagine students attending the EFL. They are there to learn more fluent English. And what language do we use to achieve that? Why English! This is because the language itself and the individual's comprehension of the language exceeds some undefined critical mass such that English can be used to talk about itself meaningfully.

Now Gödel similarly proved that any sufficiently powerful axiomatic system could also talk about itself in the language of the system. Further no matter how many axioms are used as the foundation, it is possible to construct from those axioms a further statement that can be neither proved nor disproved within the system. This must then be appended as a further axiom and so the process continues.

Here's a flavour how he did it. He used "numbers" in a one-to-one correspondence with mathematical symbols so a number might be translated into some mathematical theorem, line by line, where the commas indicate a new line. The first really clever bit was working out a method so that part of the number say the "542" bit was an encoding of the whole number. There was a further clever bit using Cantor's diagonal argument. And the whole number said mathematically "This statement cannot be proved within the system". But it could!

Finding actual examples is tricky because you're searching for the proof of something (say Fermat's Last Theorem that $a^n + b^n \neq c^n$ for $n>2$) never knowing if this happens to be something that cannot be proved. It's like searching the sock drawer for the missing sock when you don't even know if the sock is actually in the drawer or long thrown away. Fermat's last theorem was eventually proved true. That left the continuum hypothesis – how many points are there on a straight line. In fact two complementary proofs have been constructed which means there really are two different answers – termed C or $\aleph_1$. You might think that would make mathematics inconsistent. Not at all. It means you can adopt either as an additional axiom and it will not cause a contradiction in the whole system –

assuming your original axioms are actually consistent of course. Philosophers are left to argue which of the two answers might really be true if you could step outside the system. Truth really does transcend proof.

There is a curious story about Gödel's application to become a citizen of the United States. He thought he had discovered a logical flaw in the American constitution whereby a dictatorship could arise as in Germany. His sponsors were Einstein and Morgenstern. At the interview the presiding judge actually asked Gödel about this. Before Gödel got into too deep water Morgenstern steered the conversation away from this sensitive area and Gödel was duly granted citizenship.

Let's finish with Gödel's second incompleteness theorem. "No axiomatic system can prove its own consistency". So for starters we cannot ever know if our axioms actually are consistent. More problematic, if you have an inconsistent axiomatic system you can actually prove anything – from 1 + 1 = 3 upwards. That means if in our presently constructed mathematics we manage to prove it is consistent, that means it's inconsistent – because only in an inconsistent system can we prove consistency.[③]

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Mathematics and the Universe

Galileo stated "Nature's great book is written in mathematical language". Einstein remarked that the most incomprehensible aspect of the universe was its comprehensibility. Why should the universe be so ordered and subject to mathematical analysis? The answer lies partly in the "Anthropic Cosmological Principle". Because we are here to observe it, then it must be ordered. Carbon based life forms would not have evolved in a chaotic universe.

We now draw images of nature as mathematical pictures, each picture building upon the work of previous artists. Whether we ever achieve an ultimate truth is more philosophical than mathematical. Einstein's equations replaced those of Newton, which had stood for well over 200 years. Simplify

Einstein's equations and you are back to Newton. Newton was not wrong, but he could not, in his age, comprehend the next level of complexity. Einstein commented on how long his equations might last.

The next step will be to combine them with quantum mechanics to produce a comprehensive set of quantised relativistic equations. All indications are that this will be an immensely complex task. Mathematicians have to peel back continuing layers of an onion, producing ever more complex equations to explain a universe that, "at first look", seems admirably straightforward. If the ultimate truth is an incomprehensible level of complexity, how can such simplicity at the "top level" arise?

The reverse does occasionally happen. At the beginning of the twentieth century, the Scottish mathematician Maxwell reduced all that was known of electricity and magnetism into four equations. Even then, their incompatibility with the equations of motion gave Einstein his insight into relativity.

We do need to guard against the converse though. Does every mathematical concept reflect a physical reality? Professor Herbert Dingle, who long attacked the validity of Einstein's Special Theory of Relativity, commented that quadratic equations have two solutions, one of them often negative. If a real problem about the number of people required for a task gives two answers, say "plus 8" and "minus 3", we do not then immediately set out to discover the existence of minus people. We simply discard the unphysical answer.

But we need to tread carefully. Paul Dirac combined wave mechanics with relativity in one special case to produce a paired solution, one negative, and one positive. He knew the negative solution represented the electron, but after briefly considering the positive solution might be the proton, discarded it. Yet this was the first indication of the existence of the positron.

Schrödinger derived one solution to Einstein's equations of General Relativity and produced the term $(1-GM/r)$. He noted that when $r=GM$ this term, and the whole space-time metric, disappeared. He rejected the solution as unphysical. It was the English Physicist Sir Oliver Lodge who suggested there was a physical reality to the solution – the black hole. The story continues with Professor Hawking discovering that black holes "ain't that black" after all. Matter leaks out by a process similar to quantum tunnelling. Sometimes nature's laws are said to include a "cosmic censor" – dangerously unphysical situations, like time travel which is not forbidden by Einstein's equations but neither required. This is contrary to the general physical principal "that which is not specifically forbidden, will eventually happen".

The final hurdle to comprehend is the nature of mathematics itself. Rarely does anyone question its infallibility, but unfortunately it has its own inherent difficulties. One supposes that all theorems can conveniently be labelled as "true" or "false". This simple assumption is wrong. Mathematics can generate theorems that are true but cannot be proved. Further it can generate theorems that can be considered as either true or false. These can then incorporate into the whole logical structure to generate a continuing infinitude of new axioms (unprovable truths).

It seems the cosmical censor has the last laugh even in pure mathematics. What better than a system that can never be completely "solved"? In searching to prove certain theorems, mathematicians can never be sure that they aren't trying to prove the unprovable.

Sometimes they do have great successes. Fermat's Last Theorem was often quoted as a likely candidate for "true but unprovable" but this has finally succumbed to mathematical proof. There are plenty of others though– in fact a whole infinity of them – to keep mathematicians busy.

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Creation versus the Big Bang

Imagine you're clearing out your attic and come across a rusty can with a mouldy reel of film inside. You unwind the film and look at the last few frames – maybe the last hundred. That's enough to get an idea of what the film is – maybe it's your wedding. So numbering the frames backward from the last one, could you predict what would be on frame 101 by looking at the first hundred? Yes with almost exact accuracy. The same for frame 102, 103 104. There are 26 frames a second so perhaps you might be reasonably accurate to frame 150 but then it gets a bit unsure. It's after the ceremony and the bride's walking backwards up the aisle again *(you're watching the film backwards remember)* but where did she actually turn at the beginning of her walk?

Now you're a cosmologist and you hold in your hand the film of the universe – wrapped up in its reel so you can only see the last bits. You unwind the last 50 years say. Each frame is a precise description of the nature and form of the universe. Now can you predict back along the unseen parts of the film? How long is the film? And what's on that very first frame wound up tight on the inside of the reel where you can never actually see?

The universe is described in differential equations - special equations that just restate what you actually see but in a concise mathematical form. They're special because built into them is the fact that what anything does next depends on what it's doing now and how fast it's doing it. But they can be solved so you can predict back to any time in the past and know exactly what was happening. But here's the rub. To solve any differential equation you have to know or assume some initial conditions. You have to be given at least one value of x the unknown at some point in time t – not necessarily at time t = 0, any time will do. Now to the cosmologist, it's aesthetically pleasing to have the simplest initial conditions possible at time zero.

The universe is governed by Einstein's equations. There are ten of them, all intertwined, and incredibly difficult to solve but they're being chipped away year by year. Cosmologists can now run the film back 15 thousand million years and get back to a universe that's 10^{-43} seconds old (0 followed by 42 zeros then 1), composing of just a couple pounds of matter all pressed up smaller than a single atom. So the big bang is quite a neat trick. The primeval atom blows up and from that you get 100 000 million galaxies each one containing 100 000 million stars – the ultimate free lunch because quite where all that extra matter comes from is difficult to explain in plain English but mathematically a piece of cake.

But the scientist doesn't really know how long the film is because no one was around to see it. For all we know the universe was created at midnight last night and even our memories were created to that effect. The creationist and the cosmological evolutionist don't really have anything to argue about because it all boils down to an arbitrary assumption on when you start the clock.

On my first morning in Satellite Mission Analysis when I completed my apprenticeship, the senior engineer said to me "Fact 1 – the sun goes round the earth because it makes the equations simpler for earth orbital satellites". It's the same as Copernicus's bust up with the Church. In private the two parties agreed it was a pointless argument because it boiled down to whichever frame of reference you chose to base your equations on. The equations were a lot simpler if you assumed the earth went round the sun but that didn't make them the truth. Yet for the masses, that was too deep a philosophy so Copernicus had to recant in public but was allowed to continue his work in private.

Suppose God builds a house of card. He takes each one off the top of the pack carefully and places it in exactly the right position. It doesn't take Him any time to complete because He hasn't started time yet. He then realises He could have done it a simpler way. He picks up the pack, starts time, throws the cards in the air, and they all fall neatly into the right position. Which is the greater miracle? Or is it pointless to grade miracles because any miracle is still a miracle. To create two pounds of matter in one go out of nothing is no more or less miraculous than it is to create twenty trillion trillion trillion trillion trillion trillion tons of matter. To create just one atom is enough for the miracle of creation.

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The Church-Turing Hypothesis

Imagine you have a giant folder containing listings of every possible computer program. They can be written in any sensible computer language – BASIC would be fine. The only constraints are that they all start with the first line INPUT (n) – that is the program starts with inputting an integer, and they all end with the command STOP, though whether the program ever gets to it is key to the whole thesis.

You might argue such a folder would be impossible to create but you could easily employ another program, call it GAMMA, to write every one simple by creating every possible combination of allowable commands. The fact that most of the programs will be meaningless rubbish is neither here nor there.

Number the programs C_1 C_2 C_3 …. Specify what happens when each program in turn has input n, by the term $C_1(n)$ $C_2(n)$ $C_3(n)$….

The general term is thus

$C_q(n)$ is what happens when the q^{th} program is fed the number n.

Now these programs, if the list is complete, will include every possible mathematics problem. A simple one might be "Find an odd number that is the sum of n odd numbers". WE can see immediately that the program will stop when n= 1 or 3 or 5 etc. but will never stop for n = 2 or 4 or 6 – because two odd numbers will always make an even number, never another odd number. So our poor computer program is churning away trying to find an odd number that is two other odd numbers added together and never STOPping – yet we can immediately perceive the task to be hopeless.

How did we achieve our conclusion? WE used our brains which some might say is just another very complex computer program. Mind you, the problems won't all be that easy – some problems have taxed the minds of mathematicians for centuries and many are still unsolved. By unsolved we mean we don't yet know for certain if there is an answer or if it has yet been proved there is no answer.

"**No answer" equates to our program C_q(n) not STOPping.**

Clearly this is an unsatisfactory state of affairs. Computer time is valuable. There are so many other things it could be doing. We can't have it churning away forever on some particular program/problem that has no answer. We need it just to concentrate on those problems that might have a solution, so then it can churn away usefully and actually find the answer.

So we employ a group of very clever mathematicians and they sift through all the programs C_1 C_2 C_3…and all the values of n that might be inputted (n=1 n=2 n=3 etc.) putting to one side those that they can already "prove" won't STOP.

Then one day this group of mathematicians give you a very useful present. It is a computer program, call it **A**, that combines all their experience and knowledge. "This program" the leader explains, "will free us to get on with something else. Just feed in the details of each program and the value to be INPUTTED and the program **A** will tell you if C **will not stop**." Quite what **A** does if C does STOP actually need not concern us – the only condition is that, without error, **A** sifts out the non-STOPping programs so they never get activated and waste lots of valuable computing time.

Now we input two numbers into **A**, q the number of the C program and n the number to be INPUTTED. Call that **A**(q,n). So Rule (**1**)
If **A**(q,n) STOPS then C_q(n) does not stop.

Now q can have any value, so let's give it the value n. So Rule (**2**)
If **A**(n,n) stops then C_n(n) does not stop.

Now if you've followed this far, pay particular attention because for certain you'll think a trick has been played on you when you get to the end.

As **A**(n,n) is a computer program dependent on just one input n *(that's why we set q to n so it would meet this condition)* and we have already created a list of every possible computer program requiring just a single input, **so it must already be one of the C programs**! We don't know which one so just call it k for now.

So Rule (**3**) $A(n,n) = C_k(n)$

Now n can also take any value, so why not give it the value k.

So Rule (**4**) $A(k,k) = C_k(k)$

Using Rule (**2**) with n = k

If A(k,k) stops then $C_k(k)$ does not stop

But we already know that A(k,k) = $C_k(k)$ so we finally demonstrate

If $C_k(k)$ stops then $C_k(k)$ does not stop.

which is a pretty amazing conclusion by any standards.

But what does this actually mean? For certain the program $C_k(k)$ does not in fact stop, but our super computer program **A** cannot ever demonstrate this. But as WE know that $C_k(k)$ doesn't stop, we know something that **A**(k,k) doesn't know. But **A**(k,k) was supposed to encapsulate all the methods of the most brilliant mathematicians and presumably they could eventually have worked out if $C_k(k)$ STOPped or not if they'd been specifically asked. How can we know something so obvious and **A** not know it?

Where you go from here depends largely on your own prejudices, but the most common stated conclusion is that the human brain/mind cannot be reduced to simple computation *(a computer program)*, because you'll always know more than "it" knows. Also no matter how sophisticated a computer program someone else creates, you'll always be able to fool it with an input that it will churn away forever on without realising it.

Does this have a parallel in the world of computer viruses – no matter how sophisticated the virus checking program, will there always be another virus that will defeat it?

The Church-Turing thesis was first formulated purely mathematically by Alonzo Church in 1926 but reformulated by Alan Turing in terms of "Turing Machines" or more simply what today we call computers. He realised that feeding computers their own program codes would expose a limitation in any program that seemingly never bothers the human brain. Because we are conscious we can always step outside the problem and effectively say, "Ahh – you don't fool me – I see what you've done".

Where does that consciousness arise from though? Is it from the very complexity of the human brain as the Terminator films would have us believe? That is, computers will spontaneously become conscious when a certain level of complexity is reached. It's a neat idea but one that has no mathematical basis as just demonstrated in the Church-Turing thesis. Or does consciousness come from some additional element that does not follow the rules of mathematics? That suggests the X factor somehow lies outside the normal physical universe.

rg 1st March 2001

Determinism

The post Newtonian view codified by the French mathematician Laplace was that the world was deterministic. The question of how self-awareness arose and apparent free will was left for debate.

In the 20th century, two new areas of Science – Quantum Mechanics and Chaos theory, undermined determinism.

Quantum mechanics ultimately requires scientist to adopt one of four positions.

Many worlds originally put forward by Hugh Everett. In its favour it mathematically "works". Against it you seem to set aside Occams razor to postulate an almost infinite number of other universes to allow the one we live in to exist.

Faster-than-light transfer of information.

Not liked by any Einstein fan.

Super-determinism.

Even the initial conditions. Not liked by scientists because it seems to set aside free will.

Copenhagen interpretation.

There **is** no explanation.

When push comes to shove, most scientists vacillate between Copenhagen and many-worlds because they can't stand to believe in either of the other two.

Personally I don't see a conflict between super-determinism and free will. As God created time He must sit outside time and can "see" from the beginning of eternity to the end as if it were a film stretched out frame-by-frame before him. But I can watch video reruns of a football match and know what's going to happen next without taking away the original free will of the footballer (assuming he had one in the first place).

Also I think much of our problem comes from the way we approach questions. We adopt Aristotelian logic and believe in the tertium non-datum – there is no third way. Everything is black or white, is or isn't. But if you look at the mathematics of the quantum world it follows a different "quantum logic" that comes out of the mathematics without really allowing a natural explanation of what's happening. This is because quantum mechanics requires the use of imaginary numbers in probability theory and imaginary numbers don't exist.

So we argue endlessly about whether the world is A or B without ever thinking it could be both. Do you believe in Creationism or Evolution? Setting aside the fact that Darwinism is complete rubbish – and you don't need to be a Christian to know that - I personally believe both. When I'm looking at rocks on the beach I believe them to be millions of years old. When I read Genesis I believe God made the world in 7 days. Somehow they must both be true but my brain because the way it's wired isn't able to comprehend how – I just accept it – like some scientists accepting the Copenhagen interpretation of quantum mechanics – that is there isn't an explanation.

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The Fabric of Reality

A review of David Deutsch's book

What the book gains in readability is lost in believability. The starting premise is that interference patterns caused by individual photons can only be explained by the existence of the multiverse – parallel universes that interact with ours to cause the patterns.

David Deutsch quotes Hugh Everett as the author of this hypothesis to explain aspects of quantum theory, but there do seem to be key differences between Everett's and Deutsch's intepretation. Often misunderstood, Everett's many world's theory requires the splitting of the universe at each quantum measurement, but this is "universe" with a small "u" which isn't quite the same as the Universe as a whole. It would seem ludicrous to postulate that an "insignificant" measurement requires the splitting of stars billions of light years away. In fact the splitting is a local effect.

Deutsch's multiverse is never clearly described but the impression is that the infinitude of universes exists from the moment of the big bang, and thereafter diverges. Starting with infinity ensures that there are always an infinity of universes exactly paralleling ours at each quantum event. That seems to put the cart before the horse over Everett's theory. Everett starts with one universe that splits, Deutsch starts with an infinity that diverges. Further the nature and agencies of the interactions between parallel universes is never even the subject of the slightest speculation. Deutsch assumes that the interference patterns must be caused by something interfering with something else – and if the something else isn't in our universe then it must be in another.

What is often omitted though in any work that quotes Everett is that he never intended his many-worlds theory to be interpreted as a likely explanation of quantum effects. It was a convenient topic for a Ph.D. thesis identifying a possible explanation not a probable explanation. Certainly a more extreme example of ignoring Occam's razor would be hard to imagine.

But let's take a step back. Imagine that God is designing the Universe. Although an omnipotent being, He still likes to delegate the task, and one group of angels is given the brief to design "wave-mechanics". The interference patterns are duly produced and a prototype demonstration is given to God. "Have you met my original specification?" asks God. "The interference patterns are not dependent upon light intensity, are they?" "Absolutely not" replies Gabriel.

However another group has been designing the quantum and that project is also brought to a successful conclusion. It's only at a project co-ordination stage that someone asks the awkward question "Hold on a minute, how can we get an interference pattern when there is only one quanta passing through the slit?" Everyone throws up their hands (or wings) in horror but God just smiles and says "Don't worry, it'll work alright in the end".

Put it another way, the very nature of wave-particle duality, which seems central to the design of our universe, immediately requires this paradox to arise. Interference patterns are not intensity related but individual quanta will have nothing to interfere with. The Copenhagen interpretation, derided by Deutsch, is still the one most accepted by the "ordinary" scientist if pushed – don't worry about what you don't know and can never know.

There are other explanations of the quantum problem. "Many worlds" relates to the failure of contrafactual definitiveness – could things have been other than what they were? But the contrafactual aspect can fail as well as the definitiveness aspect which leads us to superdeterminism – everything including the initial conditions are absolutely fixed and free will is a myth. Mind you that sounds about as depressing as the multiverse theory. Suppose I just miss running over a child in my car. No point being elated – just be miserable thinking about all the other realities where the child actually did get knocked down.

Deutsch has a lot to say about inductive reasoning but curiously omits Peano's fifth axiom on page 223 which is the basis of mathematical induction.

That is induction cannot be proved and many mathematicians will not accept proofs based on induction. Further Deutsch on the one hand renounces the excluded middle on page 133, but gives perceptive insight into the work of Godel. You can always add another axiom <u>or its negation</u> to any consistent system – basic arithmetic being the usual candidate. So the excluded middle is required in both mathematics and quantum mechanics.

Deutsch is rather dismissive of "life" calling it an incidental scum. Yet Barrow and Tipler's monumental work "The Cosmic Anthropic Principle" gives testimony how the entire universe seems designed specifically for carbon-based life to exist. The universe is such an incredibly unlikely structure and even having "produced" the backcloth, the emergence first of life and then intelligent life which offers no evolutionary advantage are both extremely improbable events. Multiply three vanishingly small probabilities together and you have the biggest mystery of all.

rg 29th December 1998

Sliding Doors

One of Gwyneth Patrow's early films before she found Goop

You can't discount the characters' blaspheming, but try and set that temporarily aside and you have quite a good film. It's not the morality, but the moral message, and that comes over fairly well. But it's the film's basic concept, and its relation to the Christian view of God that got me thinking. And that's the idea of parallel universes.

In the film, Helen either just makes it or just misses the tube train - and from there her life takes very different though often parallel tracks.

This whole idea of alternative realities started with a bombshell dropped by Einstein and a couple of colleagues in 1935, just as the new Quantum Mechanics, the supposed theory of everything, was getting into its stride. The details are unimportant, but the upshot was that either there seemed no sensible way of understanding the universe or you had to adopt such bizarre ideas as to be unthinkable. One of these solutions, proposed by Hugh Everett in 1957, was that at each instant the universe splits in an unimaginable multiplicity of alternatives. Most physicists, if pressed, will adopt either the “no model” interpretation or, incredibly, this one. Everett himself later said he never expected the idea to be taken seriously, but that is conveniently forgotten.

Now Christians assign three particular attributes to God, one of them being omniscience - knowing everything. So that presumably means that God knows all the consequences of your actions, including those you don’t adopt. Do all these alternatives just exist in the mind of God, becoming reality as all of us make millions of decisions daily? And if God already knows what those decisions will be, how does that stack up with free will?

I had an idea that you could reconcile free will with omniscience, by imagining watching a video of someone playing chess. The chess player is, for argument’s sake, making free will decisions. But watch the video a second time - <u>now</u> you know what they will be. So the two can exist side by side.

Unfortunately, when you apply that to God, you squeeze out the second of His attributes, omnipotence. All we can do is watch the video, we can’t change it. *(God’s third attribute is omnipresence - which is how he comes to watch the chess player in the first place.)*

In fact, you can dream up little thought experiments that will capture any two of God’s attributes, while the third always manages to wriggle away. You can never capture all three together.

Which brings us back to Quantum Mechanics. The problem is you can dream up a model to capture one of reality’s attributes, but never all of them together. Light can be a wave or a particle but not both - yet it annoyingly it will be whichever you choose to measure. Ultimately Christians and Physicists *(and Christian-Physicists)* have to come to the same conclusion - that some things will forever be beyond our understanding. It’s just that Physicists can’t bear to admit it.

Footnotes

① Quite recently I put in front of my Year 3 granddaughter two equal piles of 30 beads. I then moved a smaller pile from one to the other and asked her how many beads there were in total. She started to cry and said she didn't know because she couldn't see how many I'd moved. So we spent some time moving different known piles across until she eventually said "oh it's always the same total". That was a salutary lesson for me on what or was not mathematical obvious to a girl who was in reading was about two years ahead of her biological age.

② Two mathematicians in a restaurant were arguing about the paucity of good mathematical ability among the general population. While one was on a comfort break the other briefed the waitress to say "x cubed" when asked a question. His colleague returned and the bet was laid to ask her a question – "what is the integral of $3x^2$?"

The waitress replied "x cubed". "See" said the first mathematician "people do know their mathematics". "Of course" said the waitress "but don't forget the constant".

Trust me to mathematicians this is funny

③ Some themes in this article are taken from "Riddles in Mathematics" by E. Northrop, "Escher Gödel Bach" by D. Hofstadter and "The Mathematical Experience" by Davis and Hersh.

Also available is "The Really Big Question" which is a bit more of a personal summary of my life and beliefs.