# The Relationship between Mathematics and the Physical Universe

Galileo stated "Nature's great book is written in mathematical language". Einstein took this idea a stage further. He remarked that the most incomprehensible aspect of the universe was its comprehensibility. Why should the universe be so ordered and subject to mathematical analysis? The answer lies partly in the "Anthropic Cosmological Principle". Because we are here to observe it, then it must be ordered. Carbon based life forms would not have evolved in a chaotic universe.

We now draw images of nature as mathematical pictures, each picture building upon the work of previous artists. Whether we ever achieve an ultimate truth is more philosophical than mathematical. Einstein's equations replaced those of Newton, which had stood for more than two and a half centuries. Simplify Einstein's equations and you are back to Newton. Newton was not wrong, but he could not, in his age, comprehend the next level of complexity. Einstein commented on how long his equations might last.

The next step will be to combine them with quantum mechanics to produce a comprehensive set of quantised relativistic equations. All indications are that this will be an immensely complex task. Mathematicians have to peel back continuing layers of an onion, producing ever more complex equations to explain a universe that, "at first look", seems admirably straightforward. If the ultimate truth is an incomprehensible level of complexity, how can such simplicity at the "top level" arise?

The reverse does occasionally happen. At the beginning of the twentieth century, the Scottish mathematician Maxwell reduced all that was known of electricity and magnetism into four equations. Even then, their incompatibility with the equations of motion gave Einstein his insight into relativity.

We do need to guard against the converse though. Does every mathematical concept reflect a physical reality? Certainly not. Professor Herbert Dingle, who long attacked the validity of Einstein's Special Theory of Relativity, commented that quadratic equations have two solutions, one of them often negative. If a real problem about the number of people required for a task gives two answers, say "plus 8" and "minus 3", we do not then immediately set out to discover the existence of minus people. We simply discard the unphysical answer.

But we need to tread carefully. Paul Dirac combined wave mechanics with relativity in one special case to produce a paired solution, one negative, and one positive. He knew the negative solution represented the electron, but after briefly considering the positive solution might be the proton, discarded it. Yet this was the first indication of the existence of the positron.

Schrödinger derived one solution to Einstein's equations of General Relativity and produced the term (1- GM/r). He noted that when r=GM this term, and the whole space-time metric, disappeared. He rejected the solution as unphysical. The English Physicist Sir Oliver Lodge suggested there was a physical reality to the solution – the black hole. The story continues though because Professor Hawking has now discovered, through mathematics, that black holes "ain't that black" after all. Matter leaks out by a process similar to quantum tunnelling. Sometimes nature's laws are said to include a "cosmic censor" – dangerously unphysical situations, like time travel which is not forbidden by Einstein's equations but neither required, are always avoided. This is contrary to the general physical principal "that which is not specifically forbidden, will eventually happen".

The final hurdle to comprehend is the nature of mathematics itself. Rarely does anyone question its infallibility, but unfortunately it has its own inherent difficulties. One supposes that all theorems can conveniently be labelled as "true" or "false". This simple assumption is wrong. Mathematics can generate theorems that are true but cannot be proved. Further it can generate theorems that can be considered as either true or false. These can then incorporate into the whole logical structure to generate a continuing infinitude of new axioms (unprovable truths).

It seems the cosmical censor has the last laugh even in pure mathematics. What better than a system that can never be completely "solved"? In searching to prove certain theorems, mathematicians can never be sure that they aren't trying to prove the unprovable.

Sometimes they do have great successes. Fermat's Last Theorem was often quoted as a likely candidate for "true but unprovable" but this has finally succumbed to mathematical proof. There are plenty

of others though– in fact a whole infinity
of them – to keep mathematicians busy.

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