Characteristics of the Normal Distribution Curve

The function is given as

$f(x) \quad = {}^1/_{\sqrt{2\pi}}\ e^{-\frac{1}{2}x^2}$

$\quad = {}^1/_{\sqrt{2\pi}}\ e^{(\frac{1}{2}x^2)\wedge(-1)}$

$f'(x) = {}^1/_{\sqrt{2\pi}}\ (-1)\ (e^{\frac{1}{2}x^2})^{\wedge(-2)}\ (e^{\frac{1}{2}x^2})\ (x)$

$\quad = -x\ /\ {}_{\sqrt{2\pi}\ (e^{\wedge}\frac{1}{2}x^2)}$

Hence I can see that at $x = 0$

$f'(x) = 0$ and $f(x) = {}^1/\ {}_{\sqrt{2\pi}}$

Integrating the total area under $e^{-\frac{1}{2}x^2}$ is beyond this short paper but it is $\sqrt{2\pi}$

The factor thus normalises the curve to fix the area as 1 so that the areas of individual strips can be interpreted as a probability.

Back to $f'(x) = -x\ /\ {}_{\sqrt{2\pi}\ (e^{\wedge}\frac{1}{2}x^2)}$

Let $u = -x$ so $du = -1$

Let $v = e^{\frac{1}{2}x^2}$ so $dv = e^{\frac{1}{2}x^2}\ (x)$

so $\sqrt{2\pi}\ {}^{d^2y}/_{dx^2} = (vdu - u\,dv)\ /v^2$

$\quad = \{-e^{(\frac{1}{2}x^2)} + xe^{(\frac{1}{2}x^2)}\ (x)\}\ /\ e^{x^2}$

$\quad = x^2 - 1\ /\ e^{(\frac{1}{2}x^2)}$

So I can immediately see that at $x^2 - 1 = 0$ we have ${}^{d^2y}/_{dx^2} = 0$

That is $x = \pm 1$

We know this to be correct. The point of inflexion where the second derivative is zero occurs at plus/minus 1 standard deviations.

To summarise

Maximum occurs at $(0,\ {}^1/\ {}_{\sqrt{2\pi}})$ and zero curvature occurs at $(\pm 1,\ e^{-\frac{1}{2}}/_{\sqrt{2\pi}}) \cong (\pm 1,\ 0.242)$

A quick check on the graphical calculator confirms this.

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