

Mr G's Little Book on

**Perfect and
Amicable
Numbers**

How to Find Perfect Numbers

6 is perfect because $1 + 2 + 3 = 6$ and

28 because $1 + 2 + 4 + 7 + 14 = 28$

Step 1

Write down the powers of 2.

ie start with 2 and keep doubling

2	4	8	16	32
64	128	256	512	1024
2048	4096	8192	16384	32768

Step 2

Now subtract 1 from each number

1	3	7	15	31
63	127	255	511	1023
2048	4095	8191	16383	32767

Step 3

Spot the numbers that are prime

This is a bit tricky – maybe look on google

1	<u>3</u>	<u>7</u>	15	<u>31</u>
63	<u>127</u>	255	511	1023
2047	4095	<u>8191</u>		

Now pair up these prime numbers with the previous power of 2 and multiply the two numbers together.

$$3 \times 2 = 6$$

$$7 \times 4 = 28$$

$$31 \times 16 = 496$$

$$127 \times 64 = 8128$$

$$8191 \times 4096 = 33550336$$

And these are all perfect numbers.

Commentary

How would you find that last one if you didn't know the trick? There are more but they get very big very quickly. There is a good reason why this trick works to find all EVEN perfect numbers.

That leaves open the question as to whether there is an odd perfect number. There is as yet no way either to determine one or to determine whether it exists or does not exist.

Quite a bit has been determined as to its nature if it did exist.

The smart money is that there isn't one and the smarter money is that its non existence is not provable.

How to Find Amicable Numbers

220 and 284 are amicable because

factors of 220 are 1 2 4 5 10 11 20 22
44 55 110 (220) which add to 284

factors of 284 are 1 2 4 71 142 (284)
which add to 220

There are no known rules to find all
amicable numbers only (complicated)
rules to find some of them.

The next pair (1184, 1120) wasn't
discovered until 1866 by a teenager
called Nicolo Paginini despite famous
mathematicians searching for
hundreds of years before him.

The next pair are (2620, 2924) and
(5020, 5564) but don't take my word
for it. Prove it but you can use a
calculator.

If you and your friend each carry one
number you are **guaranteed** to stay
friends for life (maybe).