General Formula for Polygon Numbers

Polygon Numbers

Let Δ_n = n^{th} triangular number which are	1	3	6	10	15	21	28
Square numbers are $\Delta_n + \Delta_{n-1}$ which are	1	4	9	16	25	36	49
Check	1	3	6	10	15	21	28
		1	3	6	10	15	21
summing gives	**1**	**4**	**9**	**16**	**25**	**36**	**49** checks
Pentagonal numbers are $\Delta_n + 2 \times \Delta_{n-1}$	1	5	12	22	35	51	70
Check	1	3	6	10	15	21	28
		2	6	12	20	30	42
summing gives	**1**	**5**	**12**	**22**	**35**	**51**	**70** checks
Hexagonal numbers are $\Delta_n + 3 \times \Delta_{n-1}$	1	6	15	28	45	66	91
Check	1	3	6	10	15	21	28
		3	9	18	30	45	63
summing gives	**1**	**6**	**15**	**28**	**45**	**66**	**91** checks

So let ε = no. sides of polygon

From direct examination of the physical patterns

$$\varepsilon_{gon} = \Delta_n + (\varepsilon - 3)\,\Delta_{n-1}$$
$$= \tfrac{1}{2} n (n + 1) + \tfrac{1}{2} (\varepsilon - 3)\, n (n - 1)$$
$$= \tfrac{1}{2} n \{ n + 1 + (\varepsilon - 3)(n - 1) \}$$
$$= \tfrac{1}{2} n (2 + \varepsilon n + \varepsilon - 2n + 2)$$
$$= \tfrac{1}{2} n \{ (2 + (\varepsilon - 2)(n - 1) \}$$

This is the general formula for the n^{th} term of an ε_{gon} number

Polygon-Centred Numbers

From direct examination of the physical patterns

Centred triangular numbers are $3 \times \tfrac{1}{2} n(n-1) + 1$

Centred square numbers are $4 \times \tfrac{1}{2} n(n-1) + 1$ etc.

and the general formula for a centred ε_{gon} is $\tfrac{1}{2} \varepsilon\, n (n - 1) + 1$