Mr G’s Little Book on

Postman Plod Labels the Babies

Introduction

A common secondary school exercise either asks children to investigate how 2, 3 4 or more babies might be labelled incorrectly in a maternity hospital or possibly how a postman might correctly or otherwise deliver letters successfully.

The consequences for a misdelivered letter may be major but more likely minor and easily corrected. The consequences in a maternity hospital are more serious.

Is there a general solution for the function $a(n)_m$ where n babies are labelled with m getting the wrong name?

Not unsurprisingly there is with some interesting discoveries along the way.

Methodology

For one baby there is only one possibly which is the right label on the right baby.

For two babies we can either get the labels the right way round or the wrong way round.

For three babies it gets more interesting. Our labels can be attached

ABC, ACB, BAC, BCA, CAB and finally CBA.

So clearly there are 3! Ways and we are dealing with permutations because the order clearly matters. Of the total permutations one will be correct, we can never have just one baby incorrect and we need to determine how many permutations have just two babies or three babies incorrect. Let's set up a table

Babies	**a**	**b**	**c**
Labels	A	B	C
	A	C	B
	B	A	C
	B	C	A
	C	A	B
	C	B	A
Correct	2	2	2

So we could tabulate the exact number correct or incorrect.

None wrong	1	One wrong	0
Two wrong	3	Three wrong	2
		Total	6

Let's repeat with four babies

Babies	**a**	**b**	**c**	**d**
Labels	A	B	C	D
	A	B	D	C
	A	C	B	D
	A	C	D	B
	A	D	B	C
	A	D	C	B

and this pattern then repeats with a leading B C and D label. In fact I initially drew up tables for up to 6 babies and searched exhaustively for all possible outcomes.

I drew up the following table

Babies	2	3	4	5	6
None wrong	1	1	1	1	1
Two wrong	1	3	6	10	15
Three wrong	0	2	8	20	40
Four wrong	0	0	9	45	135
Five wrong	0	0	0	44	264
Six wrong	0	0	0	0	265
Totals	**2**	**6**	**24**	**120**	**720**

Clearly the second row appears to be triangular numbers and there is a curious one difference – 8/9, 45/44, 264/265 – at the last non-zero pair of each column.

The sequence 2 8 20 40 is registered Sloane sequence A007290 and the function is a(n) is given by a(n–1) × n / (n–3)

The sequence 9 45 135 is registered Sloane sequence A060008 but no generating function is given.

No generating function is given by Sloane for triangular numbers but taking the clue from A007290 it is immediately obvious that a(n) is given by a(n–1) × n / (n–2).

To confirm this isn't a "one off" I can see that A060008 generating function is given by

a(n–1) × n / (n–4).

So now I have a solution for a generating function for each row but am still seeking the general solution.

So next I examined the diagonal sequence

1 2 9 44 265

These are termed recontres or derangements – permutations of n elements with no fixed points. The values are given by the Nint function – "nearest integer" so

!n = [n! / e]

The term !n is called subfactorial n.

Now as I already have the clue that each successive term can be derived from the previous term if I can build the sequence from subfactorial n - !n - I have the general solution.

So I draw up a new table

2	**8**	**20**	**40**	**70**	**112**
2×1	2×4	2×10	2×20	2×35	2×56
9	**45**	**135**	**315**	**630**	
9×1	9×5	9×15	9×35	9×63	
44	**264**	**924**	**2464**	**5544**	
44×1	44×6	44×21	44×56	44×126	

Now I can immediately see we have successive terms of $^{n}C_{m}$ where the number of babies is n and the number wrongly labelled is m. I therefore have the general solution which I modestly term Goodhand's identity

Number of ways to label n babies with m wrong (strictly n > 1)

$$a(n)_m = !n \times {}^{n}C_m$$

where !n is termed subfactorial n and is given by !n = [n! / e] the NInt function

It immediately follows that the number of ways to label AT LEAST one baby correctly is given by n! – !n

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