Power Sums

Take any number and add together each digit raised to the power n and repeat.

What happens?

After completing the investigations and armed with "solutions" of certain patterns I searched the internet to discover there are numbers called perfect digital invariants (PDI's) – number whose digits raised to a specific power sum to themselves. We always discount 1 and variants of.

Numbers that cycle round are called recurring digital invariants (RDI).

Squares

The only numbers that sum to themselves are 1 10 100 etc.

Between 1 and 100 there are 20 numbers that eventually sum to 1

1 7 10 13 19 23 28 31 32 44 49 68 70 79 82 86 91 94 97 100

eg 23 $\Rightarrow 2^2 + 3^2 = 13 \Rightarrow 1^2 + 3^2 = 10 \Rightarrow 1^2 = 1$ end

All other numbers between 1 and 100 end up in the cycle

$4 \rightarrow 16 \rightarrow 37 \rightarrow 58 \rightarrow 89 \rightarrow 145 \rightarrow 42 \rightarrow 20 \rightarrow 4$

Cubes

There are 4 PDIs - 153, 371, 370 and 407

$153 = 1^3 + 5^3 + 3^3 = 153$ $371 \Rightarrow 3^3 + 7^3 + 1^3 = 371$

$370 = 3^3 + 7^3 + 0^3 = 370$ $407 \Rightarrow 4^3 + 0^3 + 7^3 = 407$

Between 1 and 100 there are 33, 28 13 and 5 numbers respectively end up at one of these 4 numbers.

There are two 2-stage RDIs - $352 \rightarrow 160 \rightarrow 352$ and $1459 \rightarrow 919 \rightarrow 1459$.

Five and two numbers between 1 and 100 end here.

A third one is $136 \rightarrow 244 \rightarrow 136$ which I missed as nothing smaller cycles in to this.

I found one 3-stage RDI $133 \rightarrow 55 \rightarrow 250$ which 11 numbers under 100 feed into.

These are all the cubic PDIs and RDIs that exist.

Quartics

There are 2 PDIs – 1634 and 8208.

4 numbers that cycle into 8208 are 12, 17, 21 and 71.

I found two RDIs namely 6514→2178→6514 and

13139→6725→4338→4514→1138→4179→9219→13139

I believe this exhausts all possibilities.

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