

Prime Counting Function			Comparisons of relative error of the accuracy of estimates of the prime counting function												
n	$\pi(n)$	$\pi(n)/n$	$n/\ln n$	error	$\pi(n)/n/\ln(n)$	li (n)	Li (n)	li (n) - $\pi(n)$	error	rg's approx	error	$n/\ln n - 1.08366$	error	$n/\ln n - 1$	error
10	4	0.40	4	-8.57%	0.92	6.17	5.120E+00	2	-21.882%	ve log	n/a	8	ignore	8	ignore
100	25	0.25	22	13.14%	1.15	30.13	2.908E+01	5	-14.033%	3.1875E+01	-27.4987%	28	-13.59%	28	-10.95%
1000	168	16.80%	145	13.83%	1.16	177.61	1.766E+02	10	-4.850%	1.7560E+02	-4.5240%	172	-2.20%	169	-0.76%
10000	1229	12.29%	1086	11.66%	1.13	1245.87	1.245E+03	17	-1.271%	1.2393E+03	-0.8405%	1231	-0.12%	1218	0.90%
100000	9592	9.59%	8686	9.45%	1.10	9630.00	9.634E+03	38	-0.441%	9.6093E+03	-0.1801%	9588	0.04%	9512	0.83%
1000000	78498	7.85%	72382	7.79%	1.08	78628.00	7.863E+04	130	-0.167%	7.8553E+04	-0.0700%	78543	-0.06%	78030	0.60%
1.00E+07	664579	6.65%	620421	6.64%	1.07	664918.00	6.649E+05	339	-0.051%	6.6459E+05	-0.0014%	665140	-0.08%	661459	0.47%
1.00E+08	5761455	5.76%	5428681	5.78%	1.06	5.762E+06	5.762E+06	754	-0.013%	5.7605E+06	0.0163%	5768004	-0.11%	5740304	0.37%
1.00E+09	50847534	5.08%	4.83E+07	5.10%	1.05	5.085E+07	5.085E+07	1701	-0.003%	5.0840E+07	0.0156%	50917519	-0.14%	50701542	0.42%
1.00E+10	455052511	4.55%	4.34E+08	4.56%	1.05	4.551E+08	4.551E+08	3104	-0.001%	4.5500E+08	0.0123%	455743004	-0.15%	454011971	0.38%
1.00E+11	4.1181E+09	4.12%	3.95E+09	4.13%	1.04	4.118E+09	4.118E+09	11588	>0.001%	4.1177E+09	0.0090%	4.1246E+09	-0.16%	4.1104E+09	0.34%
1.00E+12	3.7608E+10	3.76%	3.62E+10	3.77%	1.04	3.761E+10	3.761E+10	38263	>0.001%	3.7605E+10	0.0068%	3.7669E+10	-0.16%	3.7550E+10	0.31%
1.00E+13	3.4607E+11	3.46%	3.34E+11	3.47%	1.04	3.461E+11	3.461E+11	108971	>0.001%	3.4605E+11	0.0052%	3.4662E+11	-0.16%	3.4562E+11	0.29%
1.00E+14	3.2049E+12	3.20%	3.10E+12	3.21%	1.03	3.205E+12	3.205E+12	314890	>0.001%	3.2048E+12	0.0041%	3.2100E+12	-0.16%	3.2014E+12	0.27%
1.00E+15	2.9845E+13	2.98%	2.90E+13	2.99%	1.03	2.984E+13	2.984E+13	1052619	>0.001%	2.9844E+13	0.0032%	2.9891E+13	-0.15%	2.9816E+13	0.25%
1.00E+16	2.7924E+14	2.79%	2.71E+14	2.79%	1.03	2.792E+14	2.792E+14	3214632	>0.001%	2.7923E+14	0.0026%	2.7966E+14	-0.15%	2.7901E+14	0.23%
1.00E+17	2.6236E+15	2.62%	2.55E+15	2.63%	1.03	2.624E+15	2.624E+15	7956589	>0.001%	2.6235E+15	0.0021%	2.6274E+15	-0.15%	2.6216E+15	0.22%
1.00E+18	2.4740E+16	2.47%	2.41E+16	2.48%	1.03	2.474E+16	2.474E+16	21949555	>0.001%	2.4740E+16	0.0018%	2.4775E+16	-0.14%	2.4724E+16	0.21%
1.00E+19	2.3406E+17	2.34%	2.29E+17	2.34%	1.02	2.341E+17	2.341E+17	99877775	>0.001%	2.3405E+17	0.0015%	2.3438E+17	-0.14%	2.3392E+17	0.20%
1.00E+20	2.2208E+18	2.22%	2.17E+18	2.22%	1.02	2.221E+18	2.221E+18	222744644	>0.001%	2.2208E+18	0.0013%	2.2238E+18	-0.13%	2.2197E+18	0.19%
1.00E+21	2.1127E+19	2.11%	2.07E+19	2.11%	1.02	2.113E+19	2.113E+19	597394254	>0.001%	2.1127E+19	0.0011%	2.1155E+19	-0.13%	2.1117E+19	0.18%
1.00E+22	2.0147E+20	2.01%	1.97E+20	2.02%	1.02	2.015E+20	2.015E+20	1932355208	>0.001%	2.0147E+20	0.0009%	2.0172E+20	-0.13%	2.0138E+20	0.17%
1.00E+23	1.9253E+21	1.93%	1.89E+21	1.93%	1.02	1.925E+21	1.925E+21	7250186216	>0.001%	1.9253E+21	0.0008%	1.9277E+21	-0.12%	1.9246E+21	0.16%
1.00E+24	1.8436E+22	1.84%	1.81E+22	1.84%	1.02	1.844E+22	1.844E+22	17146907278	>0.001%	1.8435E+22	0.0007%	1.8458E+22	-0.12%	1.8429E+22	0.15%
1.00E+25	1.7685E+23	1.77%	1.74E+23	1.77%	1.02	1.768E+23	1.768E+23	55160980939	>0.001%	1.7685E+23	0.0006%	1.7705E+23	-0.12%	1.7679E+23	0.15%
1.00E+26	1.6992E+24	1.70%	1.67E+24	1.70%	1.02	1.699E+24	1.699E+24	155891678121	>0.001%	1.6992E+24	0.0005%	1.7012E+24	-0.11%	1.6987E+24	0.14%
1.00E+27	1.6352E+25	1.64%	1.61E+25	1.64%	1.02	1.635E+25	1.635E+25	508666658006	>0.001%	1.6352E+25	0.0005%	1.6370E+25	-0.11%	1.6348E+25	0.14%

Notes RG's prime counting hypothesis *The error in the approximation $n/\ln(n)$ to $\pi(n)$ is equal to $\pi(n)/n$*

So by solving the quadratic we get $\pi(n) \approx \frac{1}{2}n\{1 - \sqrt{1 - 4/\ln(n)}\}$

so it can be seen that rg's function is more accurate than $n/\ln(n)$ but does not improve on Li(n) - sadly as expected.

In the limit my function appears to equal Chebyshev's function.

I also calculated Li (4×10^{22}) on my TI-83, but it appears the accuracy is only to 5 significant figures.