

In all the following exchanges be assured the desk sergeant always speaks the truth.

The rookie likewise except for example 6).

nb : “modus” means “the way that “ ~ ”

(I wrote this a few years back at my daughter's request before she took a police entrance exam.)

1) Proof by Hypothetical Syllogism (the theory of consequence)

PC Rookie If Tom was there so was Dick

Desk Sergeant If Dick was there so was Harry.

PC Rookie Tom was there.

Desk Sergeant Then Harry was there. Bring him in.

$[(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

2) Proof by Disjunctive Syllogism Type A (modus tollendo ponens) (~ affirms by denying)

Desk Sergeant Either Tom or Dick committed the robbery.

PC Rookie Tom was with me all evening.

Desk Sergeant Dick did it. Bring him in.

$[(p \vee q) \wedge \neg q] \Rightarrow p$

3) Proof by Disjunctive Syllogism Type B (modus ponendo tollens) (~ denies by affirming)

Desk Sergeant You and George can't have your tea break at the same time.

PC Rookie George is on his tea break.

Desk Sergeant Pick up your truncheon, son.

$[(\neg p \vee \neg q) \wedge p] \Rightarrow \neg q$

(Strictly speaking I converted $(\neg p \vee \neg q)$ to $\neg(p \wedge q)$ using de Morgan's rule. It just made it simpler to say.)

4) Proof By Detachment (modus ponendo ponens) (~ affirms by affirming) (affirming the antecedent)

Desk Sergeant If a jemmy was used it was forced entry

PC Rookie A jemmy was used

Desk Sergeant It was forced entry.

$[(p \Rightarrow q) \wedge p] \Rightarrow q$

But watch out.

Desk Sergeant It was forced entry.

PC Rookie So a jemmy was used.

Desk Sergeant No he just barged in, breaking the lock.

5) Proof By Indirect Reasoning (modus tollendo tollens) (~ denies by denying) (denying the consequent)

Desk Sergeant Whoever forged this letter will have ink on his fingers.

PC Rookie Tom doesn't have ink on his fingers

Desk Sergeant I'll have to release him then

$[(p \Rightarrow q) \wedge \neg q] \Rightarrow \neg p$

But don't jump to the false conclusion that anyone with ink on his fingers must have forged the letter.

6) Proof By Contradiction (reductio ad absurdum)

This is trickier to demonstrate by a simple exchange but try this

PC Rookie We should respect people's beliefs

Desk Sergeant Harry is a paedophile. Do you respect his beliefs

PC Rookie No of course not

Desk Sergeant So you should respect people's beliefs and not respect people's beliefs.

Perhaps you need to re-examine your original premise me lad.

In PC the compound sentence is $[(p \Rightarrow q) \wedge (p \Rightarrow \neg q)] \Rightarrow \neg p$

If assuming p is true leads to concluding both q and $\neg q$ then we must question the validity of p .

7) Proof By Cases

PC Rookie Either Tom or Dick did it that's for certain.

Desk Sergeant Tom always works with Harry

PC Rookie And Dick always works with Harry

Desk Sergeant Better bring Harry in.

$\{(p \vee q) \wedge [(p \Rightarrow r) \wedge (q \Rightarrow r)]\} \Rightarrow r$