## Godel's Proof of the Existence of God

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|---|---|
| *Axiom 1* | $\Box\ \forall x\ [\ \phi(x) \rightarrow \psi(x)\ ] \wedge P(\phi) \rightarrow P(\psi)$ |
| *Axiom 2* | $P\ (\ \neg\phi\ ) \leftrightarrow \neg P(\ \phi\ )$ |
| *Theorem 1* | $P(\phi) \rightarrow \Diamond\exists x\ [\ \phi(x)\ ]$ |
| *Defn. 1* | $G(x) \Leftrightarrow \forall\phi\ [\ P(\phi) \rightarrow \phi(x)\ ]$ |
| *Axiom 3* | $P(G)$ |
| *Theorem 2* | $\Diamond\ \exists\ x\ G(x)$ |
| *Defn. 2* | $\phi \text{ ess } x \Leftrightarrow \phi(x) \wedge \forall\Psi\{\Psi(x) \rightarrow \Box\forall x[\ \phi(x) \rightarrow \Phi(x)\ ]\}$ |
| *Axiom 4* | $P(\phi) \rightarrow \Box\ P(\phi)$ |
| *Theorem 3* | $G(x) \rightarrow G \text{ ess } x$ |
| *Defn. 3* | $E(x) \Leftrightarrow \forall\ \phi\ [\ \phi \text{ essx} \rightarrow \Box\ \exists\ x\ \phi(x)\ ]$ |
| *Axiom 5* | $P(E)$ |
| *Theorem 4* | $\Box\ \exists\ \mathbf{x}\ G(x)$ *(there exists x such that x has the property of being God)* |

| **Symbol** | **Meaning** |
|---|---|
| $P(\phi)$ | $\phi$ is a positive property |
| $G(x)$ | 'x' has the property of being God |
| $\Box$ | a necessary truth |
| $\Diamond$ | a contingent truth |
| $\rightarrow$ | implies |
| $\forall$ | for all |
| $\exists$ | there exists |
| ess x | the essence of x |
| $\neg$ | not |
| $\wedge$ | and |
| $\Leftrightarrow$ | if and only if |
| $P(E)$ | E is a positive property |

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