1) Proof by Hypothetical Syllogism (the theory of consequences)

$(p \Rightarrow q) \wedge (q \Rightarrow r)] \Rightarrow (p \Rightarrow r)$

p	q	r	p⇒q	q⇒r	(p⇒q)^(q⇒r)	p ⇒ r	[(p⇒q)^(q⇒r)]⇒(p⇒r)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

2) Proof by Disjunctive Syllogism Type A (modus tollendo ponens)(by denying affirms)

$[(p \vee q) \wedge \neg q] \Rightarrow p$

p	q	¬q	p∨q	(p ∨ q) ^ ¬q	[(p ∨ q) ^ ¬q] ⇒ p
T	T	F	T	F	T
T	F	T	T	T	T
F	T	F	T	F	T
F	F	T	F	F	T

3) Proof by Disjunctive Syllogism Type B (modus ponendo tollens)(by affirming denies)

$[\neg(p \wedge q) \wedge p] \Rightarrow \neg q$ *or write as* $[(\neg p \vee \neg q) \wedge p] \Rightarrow \neg q$

p	q	p^q	¬(p^q)	¬(p ^ q) ^ p	¬q	[¬(p ∨ q) ^ p] ⇒ ¬q
T	T	T	F	F	F	T
T	F	F	T	T	T	T
F	T	F	T	F	F	T
F	F	F	T	F	T	T

4) Proof By Detachment (modus ponendo ponens)(by affirming affirms)

$[(p \Rightarrow q) \wedge p] \Rightarrow q$

p	q	p⇒q	(p ⇒ q) ^ p	[(p ⇒ q) ^ p]⇒ q
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

5) Proof By Indirect Reasoning (modus tollendo tollens)(by denying denies)

[(p ⇒ q) ^ ¬q] ⇒ ¬p

p	q	p⇒q	¬q	(p ⇒ q) ^ ¬q	¬p	[(p ⇒ q) ^ ¬q] ⇒ ¬p
T	T	T	F	F	F	T
T	F	F	T	F	F	T
F	T	T	F	F	T	T
F	F	T	T	T	T	T

6) Proof By Contradiction (reductio ad absurdum)

[(p ⇒ q) ^ (p ⇒ ¬q)] ⇒ ¬p

p	q	¬p	p⇒q	¬q	p⇒¬q	(p⇒q)^(p⇒¬q)	[(p⇒q)^(p⇒¬q)]⇒¬p
T	T	F	T	F	F	F	T
T	F	F	F	T	T	F	T
F	T	T	T	F	T	T	T
F	F	T	T	T	T	T	T

7) Proof By Cases

{ (p v q) ^ [(p ⇒ r) ^ (q ⇒ r)] } ⇒ r

p	q	r	pvq	p⇒r	q⇒r	(p⇒r)^(q⇒r)	(pvq)^[(p⇒r)^(q⇒r)]	{(pvq)^[(p⇒r)^(q⇒r)]}⇒r
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	F	T

If we denote the Propositional Calculus statement by T ⇒ r then if we can precede it by statement p then we can produce a new tautology [p ⇒ T] ⇒ r

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