Road Capacity

In some consistent set of units let

$v =$ velocity car

$d_v =$ recommended stopping distance between cars at speed v

$g =$ length of car

$c_v =$ capacity road at velocity v

Now it is easily demonstrated that

$d_v = av^2 + bv$ where a and b are constants in some defined and consistent set of units.

To determine the capacity of the road we thus assume a distance between cars of d_v plus the length of the car g.

Now consider the time it takes for one car to pass a fixed point at velocity v

$t_v = (av^2 + bv + g) / v$ and hence

$c_v = v / (av^2 + bv + g)$

To determine the maximum value of this function we equate dc_v / dv to zero.

$dc_v / dv =$

$\{(av^2 + bv + g) - v(2av + b)\} / (av^2 + bv + g)^2$

Hence $(av^2 + bv + g) = v(2av + b)$

and rearranging we quickly deduce

$v_{max} = \sqrt{(g / a)}$

which is a pretty neat result.

As an aside the term $(av^2 + bv + g)^2$ must always be positive so we avoid spurious results where $dc_v / dv \to \infty$

Values of constants

Highway Code stopping distances are given as

mph	20	30	40	50	60
Thinking	20	30	40	50	60
Stopping	20	45	80	125	180

where distances are in feet [#].

Hence $a = 0.05$

and $b = 1$

though interestingly we don't need this value to determine v_{max}

Assume $g =$ 15 feet

$v_{max} = \sqrt{(g / a)}$

$\sqrt{(15 / 0.05)}$

17.3 mph.

As a check if we use a graphical calculator to identify the maximum value of the function $v / (0.05 v^2 + v + 15)$ and use the conversion factor 88 feet / sec = 60 mph we obtain again

$v_{max} =$ 17.3 mph again

Maximum Capacity

Inserting $\sqrt{(g / a)}$ into the function for c_v we obtain $c_{vmax} = \sqrt{(g / a)} / \{ 2g + b\sqrt{(g / a)} \}$

Inserting actual values and multiplying up to obtain maximum capacity / hour

$c_{vmax} = 60 \times 60 \times 17.3 / (2 \times 15 + 17.3)$

$\approx$ 1300 / hour

which seems a not unreasonable result.

Summary

As motorways start to "clog" the tactic is to reduce speed to increase capacity. However once a "local" speed drops below a critical value v_{max} queues will build up for no external valid reason. You join a queue, crawl for some period and then peel off the front. These queues move backwards slowly down the motorway.

an archaic form of measurement = 304.8 mm ✂ *rg*