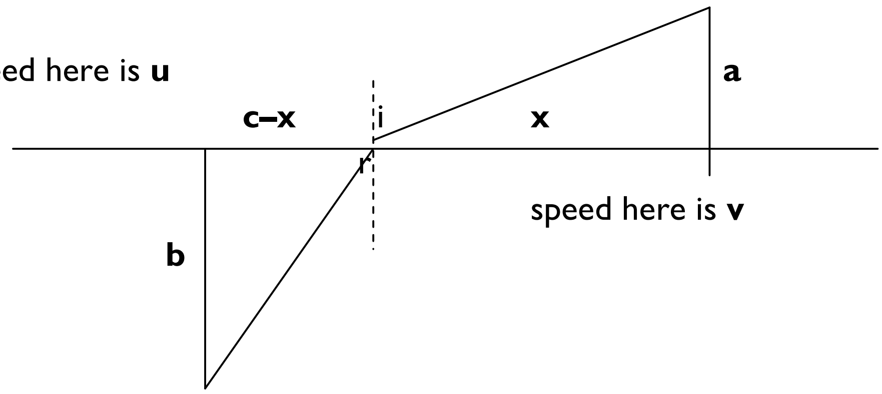
Shortest Path Problems

The debate between the corpuscular and wave theory of light was partly resolved by measuring the velocity of light in denser mediums. The simplest corpuscular theory suggests that light should travel faster in a denser medium while wave theories suggest the reverse.

Imagine two boys running toward the sea holding a rope between them. This is the wave front. They run at an angle to the sea, so one lad reaches the water first. He slows down because he now has to swim so the rope swings round as the second lad catches up. Now they are both swimming in the water, but the line perpendicular to the rope has been refracted. In fact a simple bit of geometry will actually produce the correct law of refraction $^{\sin r}/_{\sin i}$

So far so good, but how is refraction then resolved in the quantum theory of light? Fenyman proposes in his sum-over-histories formulation that the quantum in travelling from a to b seeks out <u>every conceivable path </u>even if that takes it to Mars and back. – the whole infinity of them – and then chooses the path which minimises the time (strictly the action). Whether that is really what happens is not the point here though the debate continues elsewhere and has led to great progress in the development of quantum mechanics.

So back to the original problem of finding the shortest path over two adjacent terrains. It can be immediately derived from differential calculus that the ratio of the sines of the angles at the optimum point to switch terrains is the ratio of the velocities.



time $\mathbf{t} = \quad \sqrt{[\mathbf{a}^2 + \mathbf{x}^2]} / \mathbf{u} + \sqrt{[\mathbf{b}^2 + (\mathbf{c} - \mathbf{x}^2)]} / \mathbf{v}$

so $\quad {}^{dt}/_{dx} = \frac{1}{2} [\mathbf{a}^2 + \mathbf{x}^2]^{-\frac{1}{2}}\, 2\mathbf{x} / \mathbf{u} + {}^{-}2(\mathbf{c}-\mathbf{x}).\frac{1}{2} [\mathbf{b}^2 + (\mathbf{c}-\mathbf{x})^2]^{-\frac{1}{2}} / \mathbf{v}$

at minimum value of t

$$\mathbf{x}[\mathbf{a}^2 + \mathbf{x}^2]^{-\frac{1}{2}} / \mathbf{u} = (\mathbf{c}-\mathbf{x}).[\mathbf{b}^2 + (\mathbf{c}-\mathbf{x})^2]^{-\frac{1}{2}} / \mathbf{v}$$

so $\quad {}^{v}/_{u} = [(\mathbf{c}-\mathbf{x})/\mathbf{x}] \,.\, [\mathbf{a}^2 + \mathbf{x}^2]^{\frac{1}{2}} / [\mathbf{b}^2 + (\mathbf{c}-\mathbf{x})^2]^{\frac{1}{2}}$

$= {}^{\sin r}/_{\sin i}$