An Investigation into Quadratics using TI-83

TI-83 set Zoom/ZStandard

Enter Y1 = $x^2 - 6x + 8$

Enter CALC zero and move cursor to locate two solutions

$x = 2$ and $x = 4$

This suggests we can factorise $(x - 2)(x - 4)$ and if we enter that for Y2 the graphs line up.

But let's investigate thoroughly.

Enter Y3 = x^2 and graph. Note symmetry about the y-axis which is a property of an "even" function.

Add Y3 = $x^2 + 8$.

This moves graph up 8 units and it's still symmetrical about y-axis because we can consider $8 = 8x^0$

Now change Y3= $x^2 - 6x + 8$ which repeats Y1 and Y2.

So why does adding $-6x$ move the curve diagonally to the right?

Start again with Y1 = $x^2 - 6x + 8$

Consider just the $(x^2 - 6x)$ bit.

Now $(x - 3)^2 = x^2 - 6x + 9$

So $x^2 - 6x = (x - 3)^2 - 9$

and $x^2 - 6x + 8 = (x - 3)^2 - 9 + 8$

so $x^2 - 6x + 8 = (x - 3)^2 - 1$

So we now have 3 different forms for the same function.

① $x^2 - 6x + 8$

② $(x - 2)(x - 4)$

③ $(x - 3)^2 - 1$

all identical but each giving a different insight into the nature of the curve.

Change

Y1 = x^2

Y2 = $x^2 + 8$

Y3 = $x^2 - 6x + 8$

So the basic quadratic is Y1 and the Y2 "+8" translates this 8 units up. The "–6x" translates diagonally to the left because it's a combination of

$(x - 3)$ which translates "–3" to the right and "–9" which translates 9 units down.

Why does "–3" translate positive?

On the face of it you might expect " –3 " to translate negative ie to the left.

Delete all entries and enter

Y1 = x

Observe the line passing through the origin. Now translate this line 3 down by entering

Y2 = $x - 3$ and observe. Now enter

Y3 = $(x -3)$ and observe.

This should translate the line sideways but there is no line to observe because it coincides with Y2. Obviously Y2 and Y3 are identical. A straight line translated down is indistinguishable from a straight line translated to the right.

Thus contrary to "intuition "x–3" translates to the right NOT the left.

I liken to the "Red Queen's race". You've got to add 3 just to get back to where you were.

Scaling a Quadratic

Delete all entries and enter

$Y1 = x^2 - 6x + 8$ and

$Y2 = 2x^2 - 12x + 16$

Observe that introducing or dividing through by a scaling factor will change the curve but not the zero values ie the solutions.

Solution of the Quadratic

We now have a method to find the general solution of the quadratic by algebraic means.

$$ax^2 + bx + c = 0$$

Divide through by "a" assuming $a \neq 0$

$\Rightarrow \quad x^2 + {}^{b}/_{a}\, x + {}^{c}/_{a} = 0$

Now observe $(x + {}^{b}/_{a})^2 = x^2 + {}^{2b}/_{a}\, x + \ldots$

which is double what we need. So try

$(x + {}^{b}/_{2a})^2 = x^2 + {}^{b}/_{a}\, x + {}^{b\wedge 2}/_{4a\wedge 2}$

and our first two terms now line up.

Hence

$x^2 + {}^{b}/_{a}\, x + {}^{c}/_{a} = (x + {}^{b}/_{2a})^2 + {}^{c}/_{a} - {}^{b\wedge 2}/_{4a\wedge 2}$

the last term introduced to eliminate the unnecessary term from $(x + {}^{b}/_{2a})^2$.

$\Rightarrow (x + {}^{b}/_{2a})^2 = {}^{b\wedge 2}/_{4a\wedge 2} - {}^{c}/_{a}$

which we can tidy up a bit to produce

$\Rightarrow (x + {}^{b}/_{2a})^2 = {}^{b\wedge 2}/_{4a\wedge 2} - {}^{4ac}/_{4a\wedge 2}$

Now starting to look familiar?

$\Rightarrow \quad x + {}^{b}/_{2a} = \pm\sqrt{\ (b^2 - 4ac)} / 2a$

And we finally have the familiar expression

$\Rightarrow \quad x = \{ {}^{-}b \pm\sqrt{\ (b^2 - 4ac)} \} / 2a$

Note the "x" is not strictly the same "x" in the original expression but the scaling factor "a" does not change the solutions. Although there are two solutions one of the solutions might be discarded.

Example Use of Quadratic Formula

Suppose we have $2x^2 + 5x - 12$

where $a = 2$ $b = 5$ and $c = {}^{-}12$

Hence $x = \{ {}^{-}5 \pm \sqrt{(5^2 - 4\times 2\times {}^{-}12)} \} / 4$

$\Rightarrow \quad x = \{ {}^{-}5 \pm \sqrt{(25+96)} \} /4$

$\Rightarrow \quad x = \ {}^{1}/_{4}\ ({}^{-}5 \pm 11)$

$\Rightarrow \quad x = {}^{-}4$ and/or ${}^{3}/_{2}$

and at this point you realise you could have factorised the original function into

$(x + 4) (2x - 3)$

Either bracket can be set to zero for a solution.

Even and Odd Functions

Any function can be split into even and odd terms. Investigate what common symmetry expressions like

$x^3 - 2x$ or even just $3x$ have in common

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