

Trapezoidal Numbers

The sum of any consecutive sequence is called a trapezoidal number (why?).

eg $8 + 9 + 10 + 11 = 38$

This can be considered as the difference of two triangular numbers. $T_{11} - T_7$ (why?)

Let the two triangular numbers be c and d

$$T_c - T_d = \frac{1}{2}c(c+1) - \frac{1}{2}d(d+1)$$

$$= \frac{1}{2}(c^2 - d^2) + \frac{1}{2}(c - d)$$

Which probably doesn't get us anywhere.

But tackle the problem more directly.

Take a sequence from a to b

$$a + a + 1 + a + 2 + a + 3 \dots b - 2 + b - 1 + b$$

repeat

$$b + b - 1 + b - 2 + b - 3 \dots a - 2 + a - 1 + a$$

which added together gives

$$(b - a + 1) \text{ lots of } (a + b)$$

Now complete this table

a	b	(b - a + 1)	(a + b)
even	even	?	?
even	odd	?	?
odd	even	?	?
odd	odd	?	?

So what sort of numbers can never be trapezoidal?

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