Volume of a Frustrum

A frustrum is a cone with the top chopped off. In practice it is a more commonly occurring shape than the cone eg a pile of sand or wheat is more like a frustrum than a cone. The formula for the volume is not given in the GCSE syllabus but it is implied that students should be able to calculate the volume from first principles. It's a tricky calculation. Here's two ways to find the general formula.

Method A

Let the height of a cone be t
Chop the top off and let the remaining height be h
Let the smaller radius be r
Let the bigger radius be R

To calculate the volume, you find the volume of the whole cone less the volume of the chopped off bit.
Volume $= {}^1/_3\,\pi\,R^2 t - {}^1/_3\,\pi\,r^2(t-h)$
Now from similar triangles it can be shown that $t = {}^{Rh}/_{R-r}$
Substituting back in we obtain

Volume
$= {}^1/_3\,\pi\,R^2\,({}^{Rh}/_{R-r}) - {}^1/_3\,\pi\,r^2({}^{Rh}/_{R-r} - h)$
$= {}^1/_3\,\pi\,R^3\,h/_{R-r} - {}^1/_3\,\pi\,Rhr^2/_{R-r} + {}^1/_3\pi\,r^2h$
$= {}^1/_3\,\pi\,h\,(R^3/_{R-r} - Rr^2/_{R-r} + r^2)$
$= {}^1/_3\,\pi\,h\,[R/_{R-r}\,(R^2 - r^2) + r^2]$
$= {}^1/_3\,\pi\,h\,[R/_{R-r}\,(R + r)(R - r) + r^2]$
$= {}^1/_3\,\pi\,h\,[R(R + r) + r^2]$
$= \mathbf{{}^1/_3\,\pi\,h\,(R^2 + Rr + r^2)}$

which is actually quite a neat looking formula.

Method B

Is it possible to derive the formula by a bit of logical deduction?
What is the simplest it could be?
Start with Volume $= {}^1/_3\,\pi\,h\,R^2$
Now widening out the top to a new radius r will increase the volume. We need to introduce r on equal terms to R because reversing R and r will invert the frustrum but not change the volume. So we deduce
Volume $= {}^1/_3\,\pi\,h\,(R^2 + r^2)$ *?maybe*
Setting r = 0 gives us the volume of a cone again, so that's OK.
Setting r = R should give us the volume of a cylinder.
Volume $= {}^1/_3\,\pi\,h\,2R^2$ which is incorrect.
That term ${}^1/_3$ at the beginning needs to be eliminated. So we need to add another squared term, involving R and r on equal terms. It has to be Rr.
So now we have
Volume $= \mathbf{{}^1/_3\,\pi\,h\,(R^2 + Rr + r^2)}$
Which correctly reduces to
Volume $= {}^1/_3\,\pi\,R^2h$ when r = 0 (a cone) &
Volume $= \pi\,R^2h$ when r = R (a cylinder)
So this formula is the simplest it could possibly be and give the correct results in the two limiting cases.